Math 115. Homework #13

Due Thursday 3 December

Section 7.6 (Page 344): 3, 5.

Additional problem #1
Let $\xi$ be an irrational (real) number, and let $\xi = (a_0, a_1, a_2, \ldots)$ be its expansion as an infinite simple continued fraction. Show that there exists a number $M$ such that $a_n \leq M$ for all $n$, if and only if there is a number $c > 0$ such that

$$|\xi - \frac{h}{k}| < \frac{1}{ck^2}$$

is false for all $h/k \in \mathbb{Q}$ (with $h, k \in \mathbb{Z}$ and $k > 0$, as usual).

Hints: Try “$\iff$” first. For “$\implies$”, the lemma from class on November 19 may be helpful.

Section 7.7 (Page 351): 1, 2.

Additional problem #2
Expand $\sqrt{14}$ into an infinite simple continued fraction.

Additional problem #3
In the proof of Theorem 7.21, show that $q_i > 0$ for all $i$. 