Math 113. These are Some of our Favorite Groups

This handout spells out the conventions we will always use when mentioning certain common groups.

1. When we mention $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, or $\mathbb{C}$ as groups, the group operation is ordinary addition, and the group is written additively.
2. When we mention $\mathbb{Q}^*$, $\mathbb{R}^*$, or $\mathbb{C}^*$ as groups, the group operation is ordinary multiplication, and the group is written multiplicatively. Note that $\mathbb{Q}^* = \mathbb{Q} \setminus \{0\}$, $\mathbb{R}^* = \mathbb{R} \setminus \{0\}$, and $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$.
3. When we mention $U$ or $U_n$ as groups (where $n \in \mathbb{Z}^+$), the group operation is ordinary multiplication and the group is written multiplicatively. Note that $\mathbb{Z}^+ = \{1, 2, 3, \ldots \}$.
4. When we mention $\mathbb{Z}_n$ as a group (where $n \in \mathbb{Z}^+$), its group operation is $+_n$ and it is written additively.
5. When we mention $\text{GL}_n(\mathbb{Q})$, $\text{GL}_n(\mathbb{R})$, or $\text{GL}_n(\mathbb{C})$, the group operation is matrix multiplication and the group is written multiplicatively.
6. When we mention $\text{SL}_n(\mathbb{Z})$, $\text{SL}_n(\mathbb{Q})$, $\text{SL}_n(\mathbb{R})$, or $\text{SL}_n(\mathbb{C})$, the group operation is matrix multiplication and the group is written multiplicatively.
7. We haven’t gotten there yet, but once we get to Section 8, we will define a group $S_A$ (where $A$ is any set). For this group, the group operation is $\circ$ (composition of functions) and the group is written multiplicatively.