NAME ____________________

TA's name or section number ____________________

MATH 1B Second Midterm Spring 2008

V.F.R. Jones

The first 8 questions are multiple choice, each worth 8 points. Choose the most correct answer to each question and mark the corresponding box in the grid below. Mark only one box per question. No partial credit.

Question	a	b	c	d	e
1					
2					
3					
4					
5					
6					
7					
8					

TA use only:

MC	
1	
2	
3	
4	
Total	

Multiple choice questions:

1) Which of the following is correct for any convergent series $\sum_{n=1}^{\infty} a_n$ with positive terms?

(a) $(\sum_{n=1}^{\infty} \sqrt{a_n})$ converges

(b)$\sum_{n=1}^{\infty}(a_n)^{-1}$ converges.

(c) $\sum_{n=1}^{\infty} 2^n a_n$ converges.

(d) $\sum_{n=1}^{\infty} 2^{-n} a_n$ converges.

(e) $\sum_{n=1}^{\infty} x^n a_n$ converges for all x.

2) Which of the following is correct concerning the Taylor series of $\frac{1}{1-x}$ at $a = 3$?
(a) It converges for all x.
(b)The radius of convergence is 1.
(c)The radius of convergence is 3.
(d)The function is not differentiable at $x = 1$ so the Taylor series does not exist at $a = 3$.
(e) The radius of convergence is 2.

3)If a_n is a sequence of positive numbers such that $L = \lim_{n\to\infty}(a_{n+1}/a_n)$ and $M = \lim_{n\to\infty}(a_n)^{1/n}$ both exist and are nonzero then:

(a) $L = M$ since they are both the inverse of the radius of convergence of the power series $\sum_{n=1}^{\infty} a_n x^n$

(b)$L = M^{-1}$ since they are both equal to the radius of convergence of the power series $\sum_{n=1}^{\infty} a_n x^n$

(c)$L > M$ since we don't know if the power series $\sum_{n=1}^{\infty} a_n x^n$ converges or not.

(d) L/M is the radius of convergence of the power series $\sum_{n=1}^{\infty} a_n x^n$

(e)M/L is the radius of convergence of the power series $\sum_{n=1}^{\infty} a_n x^n$

4) If $f(x)$ is a positive decreasing continuous function defined for all $x \geq 3$, which of the following is FALSE?

(a) $\sum_{n=3}^{\infty} f(n) \leq \int_3^{\infty} f(x)dx$

(b) $\sum_{n=4}^{\infty} f(n) \le \int_3^{\infty} f(x)dx$

(c)If $\sum_{n=5}^{\infty} f(n)$ converges then so does $\int_3^{\infty} f(x)dx$

(d) If $\sum_{n=6}^{\infty} f(n)$ converges then so does $\int_3^{\infty} f(x)dx$

(e)If $\int_6^{\infty} f(x)dx$ converges so does $\sum_{n=3}^{\infty} f(n)$.

5) Which of the following follows from the remainder formula in a Taylor series?

(a) The equation $e^2 = 3 + \frac{4}{2!} + \frac{8}{3!} + \frac{16e^z}{4!}$ has a solution for $1 < z < 2$

(b) The equation $e^2 = 3 + \frac{4}{2!} + \frac{8}{3!} + \frac{16e^z}{4!}$ has a solution for $0 < z < 2$

(c)The equation $e^2 = 3 + \frac{4}{2!} + \frac{8}{3!} + \frac{16e^z}{4!}$ has a solution for $z > 2$

(d) The equation $e^2 = 3 + \frac{4}{2!} + \frac{8}{3!} + \frac{e^z}{4!}$ has a solution for $0 < z < 2$

(e)The equation $e^2 = 3 + \frac{4}{2!} + \frac{8}{3!} + \frac{e^z}{4!}$ has a solution for $z > 2$

6) The radius of convergence for the Maclaurin series of $(1-x)^{10}$ is

(a) 1

(b) 0

(c) ∞

(d) 10

(e) Feeling lucky, punk?

7) Which of the following is true for a sequence $\{a_i\}$ with $a_{i+1} < a_i$ for all i?

(a)The even partial sums $\displaystyle\sum_{i=1}^{2n}(-1)^i a_i$ are increasing.

(b)The odd partial sums $\displaystyle\sum_{i=1}^{2n+1}(-1)^i a_i$ are increasing.

(c) The series $\displaystyle\sum_{i=1}^{\infty}(-1)^i a_i$ converges provided the a_i are decreasing.

(d) The series $\displaystyle\sum_{i=1}^{\infty}(-1)^i a_i$ converges provided the a_i are increasing.

(e) The limit of the sequence does not exist because its partial sums are getting bigger and bigger.

8) If the radius of convergence for the Maclaurin series of $f(x)$ is 2 we can conclude:

(a) $\lim_{n\to\infty} f^{(n)}(0) = 0$.

(b) $\lim_{n\to\infty} f^{(n)}(2) = 0$

(c) $\lim_{n\to\infty} n!f^{(n)}(0) = 0$

(d) There is an n for which $f^{(n)}(0) < n!$

(e) There is an n for which $f^{(n)}(2) = f(0)$.

The next four questions are *not* multiple choice. Show your reasoning and give your answers in the space provided.
NOTE THAT THERE ARE QUESTIONS ON BOTH SIDES OF THE PAGE!!!!!
(20 points) 1. Evaluate the following indefinite integral as a power series and give the radius of convergence:

$$\int x^2 e^{-x^2}$$

(20 points)2. Find the 20th. derivative of $f(x) = e^{x^2-2x}$ at $x = 1$, (i.e. find $f^{(20)}(1)$).

(24 points)3. Use the remainder form of a Taylor series to find out how large should n be to ensure that the errror in approximating $e^{0.5}$ by n terms in its MacLaurin series is less than 10^{-4}?

(22 points) 4. Find the Taylor series for $\sqrt{3-x}$ about $x = 2$. Give the interval of convergence not worrying about end points.