NAME ______________________________

TA's name or section number ______________________________

MATH 1B First Midterm Spring 2008

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The first 8 questions are multiple choice. Choose the most correct answer to each question and mark the corresponding box in the grid below. Mark only one box per question. No partial credit.

Question	a	b	c	d	e
1					
2					
3					
4					
5					
6					
7					
8					

TA use only:

MC	
1	
2	
3	
4	
total	

Multiple choice questions (8 points each):

1) Which of the following is correct concerning integration by parts?

(a) $\int f(x)g(x)dx = f(x)\int g(x)dx + (\int f(x)dx)\, g(x) + C$

(b) $\int f(x)g(x)dx = f(x)\int g(x)dx + (\int f(x)dx)\, g(x)$

(c) It gives the integral of any elementary function in terms of elementary functions.
(d) Integrating the product of two functions involves integrating one of them and differentiating the other.
(e)It can only be used on a product $f(x)g(x)$ if the integrals of both f and g are known already.

2) The order of the differential equation $(\dfrac{d^2y}{dx^2})^2 - \sin(\dfrac{dy}{dx}) = x^2 + 1$ is
(a) 1
(b) 2
(c) 3
(d) 4
(e) 273

3) Which of the following integrals gives the area of the surface obtained by rotating the curve $y = \ln x$ for y between 1 and 2 about the line $x = 9$?

(a)$2\pi \displaystyle\int_1^2 (9 - x)\sqrt{1 + (\ln x)^2}\, dx$

(b)$2\pi \displaystyle\int_{1/e}^{e^2} (e^x - 9)\sqrt{1 + e^{2x}}\, dx$

(c)$2\pi \displaystyle\int_e^{e^2} (9 - ln(y))\sqrt{1 + e^{2y}}\, dy$

(d)$2\pi \displaystyle\int_e^{e^2} (ln(y) - 9)\sqrt{1 + \frac{1}{y^2}}\, dy$

(e)$2\pi \displaystyle\int_1^2 (9 - e^y)\sqrt{1 + e^{2y}}\, dy$

4) Let $\{a_n\}$ be any sequence of numbers, bounded below by 4. Which of the following is correct?
(a) The sequence $\{a_n\}$ does not necessarily have a limit as $n \to \infty$.
(b) The sequence $\{a_n\}$ must have a limit as $n \to \infty$ and that limit is ≥ 4.
(c) The sequence $\{a_n - a_{n+1}\}$ tends to 0 as $n \to \infty$.
(d) The sequence $\{a_n + a_{n+1}\}$ has a limit as $n \to \infty$ and that limit is ≥ 8.
(e) The sequence $\{a_n^2\}$ has a limit as $n \to \infty$ and that limit is ≥ 16.

5) To integrate the function $\dfrac{x^4}{(x-1)(x^3-1)}$ by partial fractions one should try to express it in the form

(a) $4 + \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+x+1}$

(b) $4 + \dfrac{A}{x^3} - \dfrac{B}{x^2} + \dfrac{C}{x}$

(c) $1 + \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} - \dfrac{Cx+D}{x^2+x+1}$

(d) $1 + \dfrac{A}{(x-1)^2} + \dfrac{Bx+C}{x^2+x+1}$

(e) $1 + \dfrac{A}{x-1} - \dfrac{Bx+C}{x^2+x+1}$

6) The length of the curve $x = |y|$ for $-1 \leq y \leq +1$ is

(a) $2\sqrt{2}$
(b) 0
(c) -2
(d) $\displaystyle\int_{-1}^{1} \sqrt{1-|y|^2}\, dy$
(e) $\displaystyle\int_{-1}^{1} \sqrt{1+|y|^2}\, dy$

7) The error in estimating $\displaystyle\int_a^b f(x)\, dx$ using Simpson's rule with n intervals is at most $\dfrac{K(b-a)^5}{180n^4}$ when $|f^{(4)}(x)| \leq K$ for $a \leq x \leq b$.

Let f be a function the absolute value of whose fourth derivative is bounded by 1/9 for $2 \leq x \leq 4$. Which is the smallest of the integers $\{4, 8, 10, 12, 20\}$ for which one can be sure that the error in approximating

$$\int_2^4 f(x)dx$$

using Simpson's rule with that many intervals is less than or equal to 10^{-5}?

(a) 4
(b) 8
(c) 10
(d) 12
(e) 20

8) Which of the following is true for any sequence $\{a_n\}$ with $\lim_{n\to\infty} a_n = 3$?

(a) There is an $N > 0$ for which $a_n < 3$ for all n.
(b) There is an N for which $|a_n - 4| < 1$ for all $n \geq N$.
(c) $\lim_{n\to\infty}(a_n - a_{n+1}) = 0$.
(d) For no value of n is a_n smaller than 3.
(e)For any $\epsilon > 0$ there is an N with $|a_n - 3| > \epsilon$ for all $n \geq N$.

The next four questions are *not* multiple choice. Show your reasoning and give your answers in the space provided.

1.
a) (15 points) Evaluate the integral:

$$\int \frac{x^2}{\sqrt{9 - x^2}} dx$$

b) (15 points) Evaluate $\displaystyle\int_3^\infty \frac{dx}{x^2-4}$ or show that it is divergent.

2) (20 points) Use the comparison test to prove that one of the following improper integrals converges and one diverges. Explain your reasoning.

(a) $\int_1^{\infty} (x^3 - 1)^{-1/3} dx$

(b) $\int_0^{\pi/2} (\frac{\cos x}{x})^{1/3} dx$

3) (12 points) Is the series $\sum_{n=5}^{\infty} \frac{1}{n^2 - 3n + 2}$ convergent? If so find its sum.

4) (24 points) a)Brine with a salt concentation of 0.04 kg/litre is entering a vat at a rate of 25 litres per minute. The volume of brine in the vat is 1000 litres. If the vat initially contains pure water and the mixed brine is being drained off at 25 litres per minute, find the amount of salt in the vat as a function of time.

b) What is the name of the guy stirring the vat?