Let us find a particular solution of the differential equation coming from LCR circuits:

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = E\cos\omega t$$

Watch how sweetly it goes if we make it complex-we'll solve instead:

$$(*) \quad L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = Ee^{i\omega t}$$

and the real part of the solution is necessarily a solution to our oringal problem.

Suppose $Ae^{i(\omega t+\theta)}$ is a solution. Then differentiating and plugging in to the equation we get:

$$A\{-L\omega^2 + iR\omega + \frac{1}{C}\}e^{i\omega t} = Ee^{i\omega t}$$

So we get a solution if and only if

$$A = \frac{E}{L(\omega_0^2 - \omega^2) + iR\omega}$$

where we have used ω_0 to represent the natural frequency of the circuit without resistance, namely $\omega_0 = \sqrt{\frac{1}{LC}}$.

Now let's write the complex number A as $|A|e^{i\theta}$:

$$|A| = \frac{E}{\sqrt{L^2(\omega_0^2 - \omega^2)^2 + R^2\omega^2}}$$

and the argument is - the argument of the denominator, i.e.

$$\tan\theta = \frac{R\omega}{L(\omega^2 - \omega_0^2)}$$

So

$$\frac{Ee^{i(\omega t+\theta)}}{\sqrt{L^2(\omega^2 - \omega_0^2)^2 + R^2\omega^2}}$$

is a solution of (*) and

$$\frac{E}{\sqrt{L^2(\omega^2 - \omega_0^2)^2 + R^2\omega^2}}\cos(\omega t + \theta)$$

is a solution of our original differential equation.

Differentiating we find that the current I in the circuit oscillates with a magnitude of

$$\frac{E}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

Note the interesting fact that this is clearly a maximum when $\omega = \omega_0$ whereas the maximum amplitude for the charge oscillations occurs at a different frequency.