Here are the details of the example on surface of revolution we did not finish in class:

Find the area of the surface obtained by rotating the curve

\[ x = 1 + 2y^2 \]

for \( 1 \leq y \leq 2 \) about the x axis.

First way: as a "y" integral:

\[
2\pi \int_1^2 y \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \, dy
\]

\[
= 2\pi \int_1^2 y \sqrt{1 + 16y^2} \, dy
\]

making the substitution \( u = 1 + 16y^2 \), \( y \, dy = \frac{du}{32} \) we get

\[
2\pi \int_{65}^{17} \frac{u^{3/2}}{32} \, du
\]

\[
= \frac{\pi}{16} \left( \frac{2}{3} ((65)^{3/2} - (17)^{3/2}) \right)
\]

Second way: as an "x" integral:

\[
2\pi \int_3^9 y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx
\]

Differentiating \( x = 1 + 2y^2 \) we get \( 4y \frac{dy}{dx} = 1 \) so the integral becomes

\[
2\pi \int_3^9 y \sqrt{1 + \left(\frac{1}{4y}\right)^2} \, dx
\]

\[
= 2\pi \int_3^9 \sqrt{y^2 + \frac{1}{16}} \, dx
\]

substituting for \( y^2 \) we get

\[
2\pi \int_3^9 \sqrt{\frac{x - 1}{2} + \frac{1}{16}} \, dx
\]

\[
= 2\pi \int_3^9 \frac{1}{4} \sqrt{8x - 7} \, dx
\]

putting \( u = 8x - 7 \) we get

\[
2\pi \int_{65}^{17} \frac{1}{32} \sqrt{u} \, du
\]

AS BEFORE!!