Fourth homework assignment. Due Tuesday 26 September.

1) Show that the function on $\ell^\infty(\mathbb{N})$ defined by $||(a_n)||_\infty = \sup\{|a_n|\}$ is indeed a norm.

2) Number 31 on page 47.

3) Number 27 on p. 46

4) Number 39 p 48

5) Let $||.||_1$ and $||.||_\infty$ be the two norms on $\mathbb{R}^2$ defined in class:

$$||(x_1, x_2)||_1 = |x_1| + |x_2| \text{ and } ||(x_1, x_2)||_\infty = \max(|x_1|, |x_2|).$$

Give an explicit homeomorphism between the unit sphere in $||.||_1$ and the unit sphere for $||.||_\infty$. Show that map you define is indeed a homeomorphism. (Hint: try linear maps.)