Math 113 Quiz 1 (ungraded) January 29, 2019

Answer the questions either on the front and back of this quiz sheet or on the blank sheets provided (write your name on any additional sheets!!)

Name: ______________________________

1. What other proof-based classes have you taken (55, 104, others?)

2. The following are attempts to define functions from $\mathbb{R}$ to $\mathbb{R}$. Which of them are correctly defined functions?

(a) $f(x) = e^x$ yes

(b) $f(x) = \begin{cases} x, & x \geq 0 \\ 0, & x \leq 0 \end{cases}$ yes: both cases give a value for $x = 0$, but these values agree.

(c) $f(x) = \{y \mid y^2 = x\}$ No. $x < 0$ has ~~multi~~ no values and $x > 0$ has multiple values.

(d) $f(x) = \{y \mid y^2 = |x|\}$ No. $x \neq 0$ has multiple values.

(e) $f(x) = x^2 + 1$ yes

(f) $f(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x \leq 0 \end{cases}$ No: $f(0)$ has two values

(g) $f(x) = \begin{cases} 0, & x \geq 0 \\ 1, & x < 0 \end{cases}$ yes

3. For the purposes of this question, every cat lives either in Paris or Rome, and every flea lives on a cat. Define C to be the set of cats, F the set of fleas and $L = \{\text{Paris}, \text{Rome}\}$ the set of locations. We also define the function $h : F \to C$ from fleas to cats ("h" for host), defind by taking a flea f to the cat $h(f)$ on which it lives. We define the function $i : C \to L$ ("i" for inhabit) taking each cat to the city where it lives.

(a) Every flea lives in the same city as its host cat. Give a mathematical formula in terms of the data given for the function that takes every flea to the city wherein it lives. $i \circ h : F \to L$

(b) Write (in terms of set notation if possible) the set of cats with no fleas. many possibilities. Here's one: $\{x \in C \mid h(f) \neq x \ \forall f \in F\}$

(c) Explain in simple English what it means (for the cats and fleas) if the "host" map h is surjective. What does it mean if the map h is injective? surj: every cat has fleas. inj: every cat has at most one flea.

(d) Write a formula in set theoretic notation (as much as possible) that means that "Every cat in Paris has fleas". many possibilities. Here's one:

$\forall x \in i^{-1}(\text{Paris}), \exists f \in F$ such that $h(f) = x$

4. Remember that $\mathbb{R}^* := \mathbb{R} \setminus \{0\}$ is the set of nonzero real numbers. Define a relation on $\mathbb{R}^*$ with $a \sim b$ if $a/b > 0$ (the quotient is defined because the two elements are nonzero).

(a) What would it mean for $\sim$ to be an equivalence relation? Can you quickly check whether or not the conditions for this hold? (r): $x/x > 0 \ \forall x$ ✓ (s) if $x/y > 0$ then $y/x > 0$ ✓ (t): if $x/y > 0$ and $y/z > 0$ then $x/z = (x/y) \cdot (y/z) > 0$. ✓

(b) What are the equivalence classes of $\sim$? What is $\mathbb{R}^*/\sim$ (at least how many elements does it have, if finitely many)?

Start by looking at 1.

$[1] = \{x \in \mathbb{R}^* \mid x/1 > 0\} = \{x \in \mathbb{R}^* \mid x > 0\} = \mathbb{R}_{>0}$.

This class takes care of all positive numbers. Now look at anything outside this class. Say, $[-2] = \{x \in \mathbb{R}^* \mid x/(-2) > 0\} = \mathbb{R}_{<0}$.

So $\mathbb{R}_{>0}$ and $\mathbb{R}_{<0}$ are both ~~classes~~ equivalence classes. No other elements left, so they are the only ones.