Math 113 Homework 4, due 2/19/2019

Make sure you are using the 7th edition of Abstract algebra by Fraleigh – if you do the wrong problems, you won't get points!

1. Book exercises 5.1-5.7

2. Book exercises 5.11-5.13

3. Book exercises 5.21, 5.27, 5.28

4. Book exercise 4.28.

5. **(a)** Write down an addition table for the Klein 4-group V (look it up in the book!). Write down an addition table for the Gaussian numbers modulo 2, i.e. $\mathbb{G}/2\cdot\mathbb{G}$ (this is the group of equivalence classes in $\mathbb{G}$ modulo the relation $\equiv_2$, with $a+bi \equiv_2 a'+b'i$ if their difference is 2 times another Gaussian number, $2\cdot(c+di)$. Give a function $V \to \mathbb{G}/2\cdot\mathbb{G}$ which takes one table to the other (i.e. is an isomorphism).

(b) Recall that the direct product $\mathbb{Z}_n \times \mathbb{Z}_n$ is the group of pairs $([a],[b])$ of residues modulo n with *componentwise* addition $([a],[b])+([a'],[b'])=([a+a'],[b+b'])$. Construct an isomorphism from $\mathbb{Z}_n \times \mathbb{Z}_n$ to $(\mathbb{G}/n\cdot\mathbb{G},+)$ (here the integer $n \geq 1$ is viewed as the Gaussian number $n+0\cdot i$, and the group $\mathbb{G}/n\cdot\mathbb{G}$ is the group of residues $\mathbb{G}/\equiv_n$ where $a+bi \equiv_n a'+b'i$ if the difference is $n\cdot k$ for $k=c+d\cdot i$ a Gaussian number).

6. Extra credit, worth either 1/2 a problem or alternatively write "doing this problem instead of problem x" to replace one of problems 1.-4. but not 5: **book exercise 4.29**