Math 113 Homework 12, due 4/23/2019.
Note that the due date is the day of the midterm – it is strongly recommended that you do these exercises by Thursday, 4/18.

1. Let $f(x) = x^2 + x + 1 \in \mathbb{Z}_7[x]$. Compute the residue of $g(x) = x^4 + 3x^3 + 3x^2 + x + 4$ modulo f.

2. Let $f(x) = x^2 + 1 \in \mathbb{R}[x]$.

(a) Compute the residue of $s(x) = a + bx + cx^2$ modulo f. (It should be a polynomial of degree less than f.)

(b) Compute the product $([a + bx])([c + dx]) \in \mathbb{R}[x]/(f)$, by finding the residue of the product modulo $x^2 + 1$.

(c) Show that the function $\mathbb{R}[x]/(f)$ is isomorphic to the complex numbers via the homomorhpism $\varphi : \mathbb{R}[x] \to \mathbb{C}$ given by $\varphi([a + bx]) := a + bi$.

3. Let $g(x) = x^3 - 2x - 4 \in \mathbb{Q}[x]$. Check that $[x - 2]$ is a zero divisor in $\mathbb{R}[x]/(g)$. In other words, there is a nonzero remainder $f(x)$ (of degree smaller than g) such that $f(x) * (x - 2)$ is equal to 0 modulo $g(x)$. (Hint: use the division algorithm to divide $g(x)$ by $x - 2$.)