Math 104 Homework 9 (Vaintrob)

Due Tuesday, 4/2

1 Reading Exercise 1

Repeat example 3 (chapter 20) to (rigorously) find the derivative of $\sqrt{x}$ at π, i.e. $\lim_{x\to\pi} \frac{\sqrt{x}-\sqrt{\pi}}{x-\pi}$.

2 Reading Exercise 2

The limit of a function f at a real number a (on the boundary of its domain) exists and equals L if and only if f can be extended to a continuous function $\tilde{f}$ on $\mathrm{Dom}(f) \cup \{a\}$ with $\tilde{f}(a) = L$ (see Theorem 19.5). Using this, quickly re-prove Corollary 20.8 (without using Theorem 20.6) using the ϵ-δ notion of continuity.

3 Reading Exercise 3

(a) Suppose $f(x)$ is a continuous function on the interval $[a, b]$ and $g(x)$ is a continuous function on the interval $[b, c]$, such that $f(b) = g(b)$ (equal to some value L, say). Show that the function defined by

$$\tilde{f}(x) := \begin{cases} f(x) & x \in [a, b] \\ g(x) & x \in [b, c] \end{cases}$$

is continuous on the interval $[a, c]$.

(b) Now, look at the condition on $f(x)$ after the "if and only if" statement in Theorem 20.10 (very top of p. 161). The statement $\lim_{x\to a^-} f(x) = L$ is equivalent to saying that the function $f(x)$ to the left of a extends to a continuous function on $x \in J \mid x \leq a$, with value L at a. The statement $\lim_{x\to a^+} f(x) = L$ is equivalent to a similar statement for values to the right of a (you are allowed to use these facts). Deduce that the two conditions together are equivalent to requiring that $f(x)$ extends to a continuous function on all of J. (This line of argument can be continued to give an alternative proof of theorem 20.10.)

4 19.2

5 19.8

6 20.11

7 20.17