

Math 104 Homework 1 (Vaintrob)

Due Thursday, 1/31, reading exercises due earlier

1 Exercise 4.1, 4.2

left column: (a, c, e, g, i, k, m, o, q, s, u, w)

2 Exercise 4.1, 4.2

left column

3 Exercises 4.6, 4.7

4 Exercise 5.3

left column

5 6.1

6 Challenge problem

You can choose to do this instead of any three of the above.

Say (s_1, s_2, \dots) is a sequence of distinct points in the interval $[0, 1]$. Show that there is a “subsequence” (t_1, t_2, t_3, \dots) such that each t_i is one of the s_j and such that the t_i have a limit.

Hint: **part 1:** split the interval into two pieces $[0, 1/2]$ and $[1/2, 1]$. Since there are infinitely many distinct points s_i , there must either be infinitely many points with first binary digit 0.0 or infinitely many points with first binary digit 0.1. Make a similar argument to get infinitely many elements with the same first two digits, then three digits, etc. This lets you generate all (infinitely many) digits of a binary number, b between 0 and 1.

Part 2. Now find some subsequence (t_1, t_2, \dots) such that t_1 has the same first digit as b , such that t_2 has the correct first two digits as b , etc. Finally show that the limit of t_1, t_2, \dots is b .