Math 185 Quiz 1.

March 8, 2020

Write on the front and back of this sheet; extra blank sheets are availale.

1. (a) Find z such that $\exp(z) = -1+\sqrt{3}i$. Observe that $\arctan(-\sqrt{3}) = \pi/3$ and $|z| = \sqrt{4} = 2$, so we can use $z = i\pi/3 + \log(2)$. (Or $z + 2\pi n$ for any integer $n \in \mathbb{Z}$).

(b) Find $(-1+\sqrt{3}i)^5$. It's $\exp(10\pi i/3)\cdot 2^5 = \exp(-2\pi i/3)\cdot 2^5 = \frac{-1-\sqrt{3}i}{2}\cdot 32 = -16 - 15\sqrt{3}$.

2. $\gamma(t)$ **is a path from** $\gamma(0) = 1$ **to** $\gamma(T)$ **that satisfies** $\gamma(t) = \dot{\gamma}(t)$ **(for** $\dot{\gamma}(t) = \frac{d\gamma}{dt}$**). Compute** $\int_\gamma \frac{1}{z}dz$**.** $\int_0^T \frac{\dot{\gamma}(t)}{\gamma(t)}dt = \int_0^T \frac{\gamma(t)}{\gamma(t)}dt = \int_0^T 1 = T$.

3. You are given that $f(z) = \frac{1}{e^z+1}$ **has a power series expansion,** $\frac{1}{e^z+1} = \sum_{n=0}^{\infty} a_n z^n$ **for** $z < R$ **(radius of convergence). Prove that** R **is finite (in fact, you will probably show that** R **is less than some specific constant).** The limit of $|f(z)|$ as $z \to \pi i$ goes to infinity since $e^{\pi i} + 1 = 0$. Thus f cannot have an everywhere convergent Taylor series.