Math 185 Practice midterm 1.

March 10, 2020

1. Let $f(z) = z/(z-4)^2 + 1/(z-2i)^2$.
(a) Find the radius of convergence of this power series.
(b) Find the path integral of $f(z)$ along a circle of radius $1, 3, 5$.
(c) What is the largest radius r such that f has an antiderivative on the open disk $\mathring{D}_r$ of radius r around the origin.

2. Suppose that $f(z)$ is a function which has a singularity at $z = i$ and at $z = -i$, both of which are simple poles with residue 1. Suppose[1] that $f(z) = \int_{-5}^{5} f(x)dx = 2$.
(a) Let γ_+ be the contour $\gamma_+(t) = 5\exp(it)$ for $t \in [0, \pi]$ be the upper half-arc and let $\gamma_-(t) = -5\exp(it)$ for $t \in [0, \pi]$ be the lower half-arc of the circle of radius 5. Compute $\int_{\gamma_+} f(z)$ and $\int_{\gamma_-} f(z)$.

3. You are given that f is a function with a single singularity, which is a pole. You are also given that γ is a closed curve in $\mathbb{C}$ such that $f(\gamma(t)) = (\dot{\gamma}(t))^{-1}$ for all $t \in [0, T]$. Show that the residue of f is purely imaginary.[2]

4. Compute residues of the following functions, along the following contours.
(a) $\int_{C_2} \frac{\sin(z)}{z-i}$
(b) $\int_{C_2} \frac{e^z}{z^2}$
(c) $\int_{C_{2\pi}} \frac{1}{\cos(z)}$
(d) $\int_{C_{2\pi}} (z^2)$

[1] Originally it was assumed f is an even function... This is actually not necessary, and in fact not possible, since residues of an even function at $\pm z$ will be opposite!

[2] this question originally had a typo: f' instead of f.