Math 185 Midterm.

1

(a) Prove that $|\sin(x+iy)+\cos(x+iy)| \geq \frac{\sqrt{2}}{2}|\exp(y)-\exp(-y)|$.
(b) Give a value of z where this inequality is an equality.

2

Prove that for any nonzero polynomial P, the function $P(\exp(1/z))$ (defined on $\mathbb{C} \setminus 0$) will have an essential singularity at $z = 0$. (Hint: subtract the constant term and look at the real limits $z \to 0^+$ and $z \to 0^-$).

3

Compute the integral $\int_{-\infty}^{\infty} \frac{\exp(ix)}{(x^2+1)(x^2+4i)} dx$.

4

Prove that the function (from $\mathbb{C}$ to $\mathbb{C}$) given by $f(x+iy) = x^2+2ixy+iy^3$ satisfies the Cauchy-Riemann equations on the real line $y = 0$ but is not holomorphic.

5

Draw a simple closed curve γ such that the integral of $\frac{10^{\frac{z}{\pi}}}{\sin(z)}$ along γ is $2\pi i \cdot 90.009$ (Here the function 10^z is defined as $\exp(z \cdot \log(10))$.

6

Find the Laurent series expansion at 0 of the function $f(z) = \frac{1}{\exp(z)+i\exp(iz)-1-i}$ up to the z^1 term. What order of singularity is this pole? Compute the residue of f at 0.