Math 141 Homework 2. Due 9/17

"GPn" denotes Guillemin-Pollack, chapter 1, section n. Scan of exercises provided in `www.math.berkeley.edu/~vaintrob/141/gp_hw.pdf`

You may do problem 3 (GP2, 1 and 2) as part of the next homework for full points, as we have not covered differentials of maps between manifolds yet.

1. GP1 6, 7

2. GP1 18

3. GP2, 1, 2

4. GP2, 6, 7. Correction: problem 18 (b) should read "g(x) = f(x-a)f(b-x)."

5. Suppose $X \subset \mathbb{R}^N$ is a manifold and suppose $f : \mathbb{R}^N \to \mathbb{R}$ is a smooth function such that $f(p) = 0$ for all $p \in X$. Prove that for all $p \in X$,

$$T_p(X) \subset \operatorname{Ker}\big(df(p)\big)$$

(here Ker denotes kernel of a linear operator). For $X = S^{n-1} \subset \mathbb{R}^n$, find with proof a function f such that $f(p) = 0$ for $p \in S^{n-1}$ and

$$T_p(S^{n-1}) = \operatorname{Ker}\big(df(p)\big)$$

for all $p \in S^{n-1}$.