Math 185 Complex Analysis, Fall 2017. Instructor: Dmitry Tonkonog

Midterm 2

Duration: 60 min

No books, notes or calculators are allowed. A blank sheet of paper is provided at the end of the exam. If you need more paper, please get it from the proctor. In the case of a fire alarm, leave your exams in the room, face down, before evacuating. Under no circumstances should you take the exam with you.

This midterm has 5 problems. You are welcome to refer to all general results from Chapters 1–5 from the textbook of Brown and Churchill, 9th ed. You are asked *not* to use the results from Chapters 6 (Residues and Poles) and further chapters.

Your name:

1:___/12 2:___/12 3:___/12 4:___/8 5:___/16 Σ:___/60

Problem 1 (12 points). Let $\sum_{n=0}^{\infty} a_n z^n$ be the Taylor expansion of the function

$$f(z) = e^{z^2} \sinh(z^3)$$

at the origin. Find the coefficients a_5 and a_9.

Problem 2 (12 points). Consider the function

$$f(z) = \frac{1}{z^2 - 5z + 6}.$$

Find its Taylor expansion at the origin and state its radius of convergence.

Hint: it holds that $z^2 - 5z + 6 = (z - 2)(z - 3)$.

Problem 3 (12 points). Consider the Taylor expansion $\sum_{n=0}^{\infty} a_n z^n$ of the function

$$f(z) = \frac{z^3 - 2}{\sin(\pi/2 + \pi \cos z)}$$

at the origin. Does that Taylor series converge for $z = 2$? Justify your answer. (You are not required to compute the Taylor expansion explicitly.)

Problem 4 (8 points). Compute the integral

$$\int_{|z|=10} \frac{e^z}{(z-2\pi i)^4} dz.$$

The countour is oriented counterclockwise.

Problem 5 (16 points). Compute the following integrals:

$$\int_C \frac{e^z}{z(1-z)^3} dz$$

where:

(a) $C = \{|z| = 1/2\}$,
(b) $C = \{|z - 1| = 1/2\}$
(c) $C = \{|z| = 3/2\}$.

All contours are parametrized counterclockwise.

[Blank page]