Math 185 Complex Analysis, Fall 2017. Instructor: Dmitry Tonkonog

Midterm 1

Duration: 60 min

No books, notes or calculators are allowed. A blank sheet of paper is provided at the end of the exam. If you need more paper, please get it from the proctor. In the case of a fire alarm, leave your exams in the room, face down, before evacuating. Under no circumstances should you take the exam with you.

This midterm has 6 problems.

Your name:

1:___/8 2:___/8 3:___/12 4:___/12 5:___/12 6:___/8 Σ:___/60

Problem 1 (8 points). On the complex plane, draw the set of complex numbers z satisfying

$$|z + i| = |z + 3i|.$$

Problem 2 (8 points). Compute the norm and the argument of

$$(1+i)^{15}.$$

Your answer regarding the argument must be a number within $[0, 2\pi)$. Show on a picture which quadrant of the complex plane contains the number $(1+i)^{15}$.

Problem 3 (12 points). Let $z_1, z_2, z_3 \in \mathbb{C}$ be three pairwise distinct complex numbers. Let us write each of these numbers in Cartesian form: $z_k = x_k + iy_k$, $k = 1, 2, 3$. Prove that the three points on the plane

$$(x_k, y_k) \in \mathbb{R}^2, \quad k = 1, 2, 3,$$

lie on the same line if and only if

$$\frac{z_3 - z_1}{z_2 - z_1}$$

is a real number.

Problem 4 (12 points). Recall that a polynomial $P(z)\colon \mathbb{C} \to \mathbb{C}$ is a function of the form

$$P(z) = a_0 + a_1 z + \ldots + a_n z^n, \quad a_i \in \mathbb{C}.$$

Suppose $P(z)$ is a polynomial satisfying

$$P(z) = \overline{P(\bar{z})}$$

for all z. Without using the reflection principle, prove that its coefficients a_i are all real.

Problem 5 (12 points). Prove that

$$\lim_{z\to 0} z e^{\bar{z}/z} = 0.$$

Problem 6 (8 points). Find all points on the complex plane where the function $f(z)\colon\ \mathbb{C} \to \mathbb{C}$ given below is differentiable:

$$f(z) = 3xy - i(x^2 - y^2), \quad z = x + iy.$$

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