Math 185 Complex Analysis, Fall 2017. Instructor: Dmitry Tonkonog

Practice Midterm 2

The midterm will be 60 min long, no notes allowed.

1. Let $\sum_{n=0}^{\infty} a_n(z-\pi i)^n$ be the Taylor expansion of

$$f(z) = \sinh z$$

at the point $z_0 = \pi i$. Find the coefficients a_3, a_4.

2. Using the Taylor expansion for $1/(1-z)$, find the Taylor expansion of

$$\frac{1}{(1+z^2)^2}$$

at the origin and state its radius of convergence.

3. Suppose a Taylor series $\sum_{n=0}^{\infty} c_n z^n$ converges for all z such that $|z| \le R$. Prove that there exists a number $M \in \mathbb{R}$ such that $|c_n| < M/R^n$ for all n.

4. Compute the following integral:

$$\int_{|z|=5} \sin z \left(\frac{1}{z^4} - \frac{1}{(z-\pi)^4} \right) dz.$$

The countour is oriented counterclockwise.

5. Compute the following integral:

$$\int_{|z+1|=1} \frac{dz}{(1+z)(z-1)^2}.$$

The contour is oriented counterclockwise.

Answers can be found on the next page.

Answers:

1. $a_3 = -1/6$, $a_4 = 0$.

2. $\sum_{n=0}^{\infty}(n+1)(-1)^n z^{2n}$, $|z| < 1$

3. Solution: take the real point $z = R$, then the sum becomes $\sum c_n R^n$. We are given that it converges, so its terms must tend to zero: $c_n R^n \to 0$ as $n \to +\infty$. The terms of a convergent sequence are bounded, so there exists an M such that $|c_n R^n| < M$ for all n. Equivalently, $|c_n| < M/R^n$ for all n.

4. $-2\pi i/3$.

5. $\pi i/2$.