

*Practice Midterm 1*

*The midterm will be 60 min long, no notes allowed.*

**Problem 1.** On the complex plane, draw the set of complex numbers satisfying

$$|z - 1| = |\operatorname{Re} z|.$$

**Problem 2.** Find all solutions of the equation

$$e^z = \frac{1}{2}(1 - i).$$

**Problem 3.** Let  $z_1, z_2, z_3 \in \mathbb{C}$  be three pairwise distinct complex numbers. Let us write each of these numbers in Cartesian form:  $z_k = x_k + iy_k$ ,  $k = 1, 2, 3$ . Prove that the three points on the plane

$$(x_k, y_k) \in \mathbb{R}^2, \quad k = 1, 2, 3,$$

are the vertices of an equilateral triangle if and only if

$$(z_3 - z_1)(z_2 - z_3) = (z_1 - z_2)^2$$

and

$$(z_1 - z_2)(z_3 - z_1) = (z_2 - z_3)^2.$$

**Problem 4.** Show that

$$\lim_{z \rightarrow \infty} \frac{a_0 + a_1 z + \dots + a_n z^n}{b_0 + b_1 z + \dots + b_n z^n} = \frac{a_n}{b_n},$$

where  $a_i, b_i \in \mathbb{C}$  are fixed numbers and  $b_n \neq 0$ .

**Problem 5.** Show that the limit

$$\lim_{z \rightarrow \infty} e^{-z}$$

does not exist.

**Problem 6.** Find all points on the complex plane where the function

$$f(z) = z \operatorname{Re} z$$

is differentiable.