1MA259 Differential Topology, Spring 2017

Exercises to Lectures 16-18

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1. Show that the antipodal map $S^n \to S^n$ preserves orientation if and only if n is odd. Conclude that the (integral) degree of the antipodal map equals $(-1)^{n-1}$.

2. Construct a homotopy between the antipodal map $f\colon S^{2n-1} \to S^{2n-1}$ and the identity map.

3. Prove that the Mobius band embedded in $\mathbb{R}^3$ cannot be contained in a regular level set of some function $f\colon \mathbb{R}^3 \to \mathbb{R}$.

4. Suppose X a manifold which is not orientable. Prove that $X \times Y$ is not orientable, for any Y.

5. Let $f(z) = 1/z$ on the circle of radius r in $\mathbb{C}$. Compute $\deg(f/|f|)$. Why does our proof of the Fundamental Theorem of Algebra not imply that $1/z$ has a root in $\mathbb{C}$?

6. Prove that every map $S^1 \to S^1$ is homotopic to the map $z \mapsto z^k$, $|z| = 1$, for some $k \in \mathbb{Z}$.

7. Explain why the definition of Euler characteristic via the self-intersection number of the diagonal actually works for non-orientable manifolds, as well. (See Guillemin-Pollock, p.118-119 for details.)

8. Prove that $\chi(X \times Y) = \chi(X) \cdot \chi(Y)$.