1MA259 Differential Topology, Spring 2017
Exercises to Lectures 13-14
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1. Construct a homotopy between the antipodal map $f\colon S^{2n-1} \to S^{2n-1}$ and the identity map.

Hint: do this for S^1 first, and then break up the co-ordinates on $\mathbb{R}^{2n}$ into pairs.

2. Let $T^2 = S^1 \times S^1$ be the torus; it contains two loops called the *meridian* and the *longtitude*. Construct a natural bijection

$$\{\text{maps } f\colon S^1 \to T^2\}/\textit{homotopy} \to \mathbb{Z}^2$$

by showing that every such f is homotopic to a loop which goes a times along the meridian and b times along the longtitude. A more precise formulation is left to you.

3. The little prince and the sheep live on a planet whose surface is T^2. They once travelled across the planet and returned to their original locations (which are distinct). The planet has two rivers, flowing along the meridian and the longtitude. The little prince crossed the meridian 9 times and the parallel 6 times, while the sheep—8 and 7 times, correspondingly. Prove that the routes of the little prince and the sheep intersected.

Hint: use intersection theory to get information about their routes seen as elements of $\mathbb{Z}^2$, as in the previous exercise.

4. Suppose now the little prince and the sheep live on the Klein bottle. It also has a meridian and a longtitude. In the same setup as above, must the routes of the little prince and the sheep intersect?