MATH 54 Quiz 1: Solutions

Problem 1 (5 points). Suppose that each of the following matrices is the reduced echelon form of the *augmented* matrix of a system of linear equations. In each case, determine if the system is consistent. If it is consistent, determine if the solution is unique.

$$\text{(a)} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \text{(b)} \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 3 \end{bmatrix} \qquad \text{(c)} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

Solution:

(a) Consistent. This is because there are no pivots in the last column. The solution is not unique. This is because the bottom row is all zeroes, so one of our variables is free (we basically have 2 equations and 3 variables)

(b) This is consistent because there are no pivots in the last column. It has unique solutions since there is no row of all zeroes; all our variables are uniquely determined.

(c) This is inconsistent because there is a pivot in the last column.

Problem 2 (5 points). Let $\mathbf{v}_1 = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$.

(a) Compute $\mathbf{v}_1 + \mathbf{v}_2$ and $3\mathbf{v}_1 - \mathbf{v}_2$.

(b) Is $\mathbf{b} = \begin{bmatrix} 10 \\ 1 \end{bmatrix}$ a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$?

Solution:

(a)

$$\mathbf{v}_1 + \mathbf{v}_2 = \begin{bmatrix} 4 \\ -1 \end{bmatrix} + \begin{bmatrix} 6 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 \\ 1 \end{bmatrix}$$

$$3\mathbf{v}_1 - \mathbf{v}_2 = \begin{bmatrix} 12 \\ -3 \end{bmatrix} + \begin{bmatrix} -6 \\ -2 \end{bmatrix} = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$$

(b) Yes, in particular $\mathbf{b} = \mathbf{v}_1 + \mathbf{v}_2$.

Problem 3 on back →

Problem 3 (5 points). Let $\mathbf{a}_1 = \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} 1 \\ 7 \\ 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 22 \\ -1 \\ 6 \end{bmatrix}$. Is $\mathbf{b}$ a linear combination of $\mathbf{a}_1, \mathbf{a}_2$, and $\mathbf{a}_3$?

Solution: Yes, $3\mathbf{a}_1 + 2\mathbf{a}_2 - \mathbf{a}_3 = \mathbf{b}$. To figure this out, we can row reduce the augmented matrix for the associated system of equations a bit.

$$\left[\begin{array}{ccc|c} 5 & 4 & 1 & 22 \\ 0 & 3 & 7 & -1 \\ 1 & 2 & 1 & 6 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|c} 0 & -6 & -4 & -8 \\ 0 & 3 & 7 & -1 \\ 1 & 2 & 1 & 6 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|c} 0 & 0 & 10 & -10 \\ 0 & 3 & 7 & -1 \\ 1 & 2 & 1 & 6 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|c} 0 & 0 & 10 & -10 \\ 0 & 3 & 7 & -1 \\ 1 & 2 & 1 & 6 \end{array}\right]$$

And at this point we have enough info to make solving the problem easy. We know that $10c_3 = -10 \implies c_3 = -1$. Next, $3c_2 + 7c_3 = -1 \implies 3c_2 = -1 - 7(-1) = 6 \implies c_2 = 2$. Lastly, $c_1 + 2c_2 + 1c_1 = 6 \implies c_1 + 4 - 1 = 6 \implies c_1 = 3$. And we have found the linear combination that we stated at the beginning.