General Section Notes

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1 Thoughts on Quiz 3.

Problem 1 had a few common mistakes. The mistakes were something like describing the taste of something as "red." The adjective "red" does not apply to the noun "taste."

1. A is onto/one-to-one. Why is this a mistake? Onto and one-to-one are descriptions of **maps**. A on its own is a matrix, not a map. The correct statement would be that the map $\mathbf{x} \mapsto A\mathbf{x}$ is onto/one-to-one.
2. A is linearly independent. Linearly independent is a description of a **set of vectors**. A on its own is a matrix, not a set of vectors. The correct statement would be that the columns of A are linearly independent (or the rows of A).

2 Thoughts on Quiz 4.

The first question stumped a lot of people. If you are having trouble with the idea of a **standard matrix**, you need to figure it out! See this ed post and use your resources to learn this before the midterm.

The second question had two common mistakes.

1. "0 is not in H, so it is not a subspace." If it were true that 0 is not in H, then the conclusion would be correct. However, the problem explicitly states that 0 is included in H. The condition that 0 is in H is a bit finnicky – from any set H, one could simply add or remove the point 0. Sometimes verifying that 0 is not in H is a quick way to check that something isn't a subspace, but it's not the most important condition.
2. "0 is in H, so it is a subspace." To be a subspace, H has to satisfy **3** conditions. That means you would have to show that H satisfies all **3** conditions to show that it is a subspace. Checking one condition is not enough.

3 Thoughts on Homework 5.

Please, please, please **think deeply** about your homework. It was clear that a lot of people avoided thinking about problems 4.2.23 and 4.3.40. If you do not think about the problems that you don't understand, there is no point in doing them. Your task as a learner is to figure out things that you don't understand. Sometimes this is hard work. It's like eating your vegetables, you may not want to do it, but it's good for you (credit to Galen Liang for this simile).

What does it look like to think deeply about problem 4.2.23? Let's see.

Problem 3.1. Assume H and K are subspaces of V. Show that $H \cap K$ is a subpace of V.

1. We need to know what we are trying to do. This means knowing the definitions of the words and symbols in the problem. Whenever you don't know what a word in math means, **find the definition**. Don't invent a meaning for the word. Don't skip the word.

   If you look in your textbook, you will find the following definition.

   Definition 3.2. Let V be a vector space. A subset S of V is a vector space if and only if

   i. The zero element of V is in S, i.e. $\mathbf{0} \in S$.

   ii. For any $\mathbf{u} \in S$ and $\mathbf{v} \in S$ in S, $\mathbf{u} + \mathbf{v} \in S$.

   iii. For any $\mathbf{u} \in S$ and $c \in \mathbb{R}$, $c\mathbf{u} \in S$.

   Note: $x \in S$ is spoken "x in S" or "x is an element of the set S."

   If you use one of your resources, you will also find this definition.

   Definition 3.3. Let A and B be sets. The intersection of A and B, written $A \cap B$ is the set of all things that are both in A **and** in B. Written in symbols,

   $$A \cap B = \{x : x \in A \text{ and } x \in B\}.$$

2. Now that we have our definitions, we have to **try** using them. We need to show $H \cap K$ is a subspace. This means we have to verify the 3 conditions in definition 3.2.

   i. Is $0 \in H \cap K$? By definition 3.3, $0 \in H \cap K$ if $0 \in H$ **and** $0 \in K$.

   We know (from the problem) that H and K are subspaces. By definition 3.2, this means that $0 \in H$ and that $0 \in K$. Since $0 \in H$ **and** $0 \in K$, we know that $0 \in H \cap K$.

   We conclude that $H \cap K$ satisfies the part (i) of the definition of a subspace.

   ii. Suppose $\mathbf{u} \in H \cap K$ and $\mathbf{v} \in H \cap K$. Is $\mathbf{u} + \mathbf{v} \in H \cap K$?

   Since $\mathbf{u}$ and $\mathbf{v}$ are in $H \cap K$, we know that $\mathbf{u}$ and $\mathbf{v}$ are both in H. Since H is a subspace, we know that $\mathbf{u} + \mathbf{v} \in H$.

   Since $\mathbf{u}$ and $\mathbf{v}$ are in $H \cap K$, we also know that $\mathbf{u}$ and $\mathbf{v}$ are both in K. Since K is a subspace, we know that $\mathbf{u} + \mathbf{v} \in K$.

   Since $\mathbf{u} + \mathbf{v} \in H$ **and** $\mathbf{u} + \mathbf{v} \in K$, we know that $\mathbf{u} + \mathbf{v} \in H \cap K$. We conclude that $H \cap K$ satisfies part (ii) of the definition of a subspace.

   iii. Verify condition (iii) yourself.

If you didn't think about problem 4.3.40, **revisit it** and think deeply about it. Try the strategy we just used for problem 4.2.23. First find all the definitions, then work on answering the question using those definitions. Take it one step at a time.

4 Some Review Problems

Please start reviewing for the midterm ASAP! Here are a couple of problems I mentioned in class on 03/15 that cover some material from weeks 2-3. Credit to Robert Schütz for these.

Problem 4.1. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & 6 & 8 \\ 2 & 3 & 4 \end{bmatrix}$$

Is there a sequence of row operations that can transform A into B?

Problem 4.2. This problem is about span.

1. Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in $\mathbb{R}^n$. Prove that $\text{Span}(\{\mathbf{u}, \mathbf{v}\}) = \text{Span}(\{\mathbf{u}, \mathbf{u} - \mathbf{v}\})$.
2. Let $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ be vectors in $\mathbb{R}^3$. Suppose that $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ span $\mathbb{R}^3$. Define $A = \begin{bmatrix} \mathbf{u} & \mathbf{v} & \mathbf{w} \end{bmatrix}$. Prove that for any $\vec{\mathbf{b}} \in \mathbb{R}^3$, $A\mathbf{x} = \vec{\mathbf{b}}$ is consistent.