Math 53 Dis 108/111 Worksheet 9 (10/11)

Tom Schang

Problem 1. Consider the triangle with vertices at $(0,0)$, $(0,1)$, and $(1,1)$.

(a) Draw the triangle.

(b) Compute the area of the triangle without using any calculus.

(c) Compute the area of the triangle by using a double integral. Verify that you get the same answer.

(d) Compute the area of the triangle by again using a double integral, but exchange the order of integration.

Problem 2. For each of the following, draw the region described by the equations, then compute the area of the region.

(a) The region between the curves $y = x$ and $y^2 = x$.

(b) The region enclosed by $x = 0$, $x = 2y$ and $x = 3y - 1$.

(c) The region where $1 \leqslant xy \leqslant 2$ and $2 \leqslant x \leqslant 4$.

(d) The region where $1 \leqslant xy \leqslant 2$, $0 \leqslant y \leqslant 2$ and $0 \leqslant x \leqslant 2$.

(e) The left half of the unit circle.

(f) The region between the unit circle and $y = x^2$.

Problem 3. For each of the following, draw the region described by the integral, then describe the region using equations/inequalities.

(a)
$$\int_0^2 \int_1^{1+x^2} \mathrm{d}y\,\mathrm{d}x$$

(b)
$$\int_1^{\pi} \int_{1-x}^{\sqrt{x^2-1}} \mathrm{d}y\,\mathrm{d}x$$

(c)
$$\int_{-1}^{0} \int_{y^2}^{\sqrt{-y}} \mathrm{d}x\,\mathrm{d}y$$

Problem 4. For each problem in (2) and (3), exchange the order of integration.

Problem 5. Compute $\int_0^1 \int_{3x}^3 \cos(y^2)\,\mathrm{d}y\,\mathrm{d}x$. [Hint: problem 1 (d)].

Problem 6. (Credit to Xiaoke Song) Compute the following integrals.

1. $D = \{1 \leqslant y \leqslant 2, y \leqslant x \leqslant y^3\}$.
$$\iint_D e^{\frac{x}{y}} \, \mathrm{d}A$$

2. D is the region enclosed by $y = x^3$ and $y = \sqrt{x}$.
$$\iint_D 4xy + y^3 \, \mathrm{d}A$$

3.
$$\int_0^1 \int_{x^2}^1 \sqrt{y} \sin y \, \mathrm{d}y \, \mathrm{d}x$$

4. [Hint: problem 3, and then don't actually compute the integral. Figure out what it has to be because of what the integral means].
$$\iint_{x^2+y^2 \leqslant 1} \sqrt{1 - x^2 - y^2} \, \mathrm{d}A$$