Math 53 Dis 108/111 Worksheet 8 (10/02)

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Problem 1. Maximize $f(x, y, z) = xyz$ on the plane $x + y + z = 10$ in two ways

1. Without Lagrange multipliers (eliminate a variable).
2. By using Lagrange multipliers.

Problem 2. Find the maximum and minimum distance from the point $x = 2, y = 1$ to the unit circle $x^2 + y^2 = 1$ in three ways

1. Parametrize the circle.
2. Eliminate a variable.
3. Use Lagrange multipliers.

Observe the different things you have to check when using each of these methods. Which of these methods would translate easily if instead we were in 3d (i.e. we were finding distance to a unit sphere?)

Problem 3. Suppose Jasper is a dog who lives in a fenced in region R of the plane given by

$$R = \{(x, y) \in \mathbb{R}^2 : x^2/4 + y^2 = 1\}.$$

Moreover, there are two lemons, which smell really badly to Jasper, nearby. One is at the point $x = 3$, and the other is at $x = -3$. Suppose the strength of the combined smell of lemons at a point x, y is

$$f(x, y) = \frac{1}{(x-3)^2 + y^2} + \frac{1}{(x+3)^2 + y^2}.$$

Jasper wants to stand in the place in his enclosure where the smell is weakest. Where should he stand?

(a) Write down a step-by-step plan for how to solve this problem (using Lagrange multipliers). You must explicitly write out any equations that must be solved in terms of x and y, but you don't have to solve those equations.

(b) How would you change your step-by-step plan if instead Jasper lived in the region

$$S = \{(x, y) \in \mathbb{R}^2 : x^2/4 + y^2 = 1 \text{ and } y \leqslant 1/3\}?$$

Be explicit, just as in the previous question.

(c) Jasper has a really good nose, and he can smell the instantaneous rate of change of strength smell in any direction. Jasper smells no instantaneous change in the strength of smell in any direction when he stands at $(0, 0)$. He thinks he has found the spot where the smell is weakest. Convince Jasper that he is wrong.

Problem 4. Consider the region inside the unit ball in $\mathbb{R}^3$ (i.e. $x^2 + y^2 + z^2 \leqslant 1$) and that satisfies $x + y + z \geqslant 1$.

1. Find the maximum and minimum distance between this region and the point $(0, 0, 1/4)$.
2. Now, suppose instead that $x + y + z \geqslant a$. For each possible a, what is the maximum and minimum distance?

Problem 5. Consider the 3d region inside the cylinder $x^2 + y^2 \leqslant 4$ and outside of the hyperboloid $y^2 - x^2 - z^2 \geqslant 1$. Find the maximum of $f(x, y, z) = xz - y$ on this region.

Problem 6. Explain how Lagrange multipliers with two (or generally multiple) constraints works/why it finds critical points of f subject to those constraints.