Math 53 Dis 108/111 Worksheet 6 (09/13)

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1 Partial Derivatives

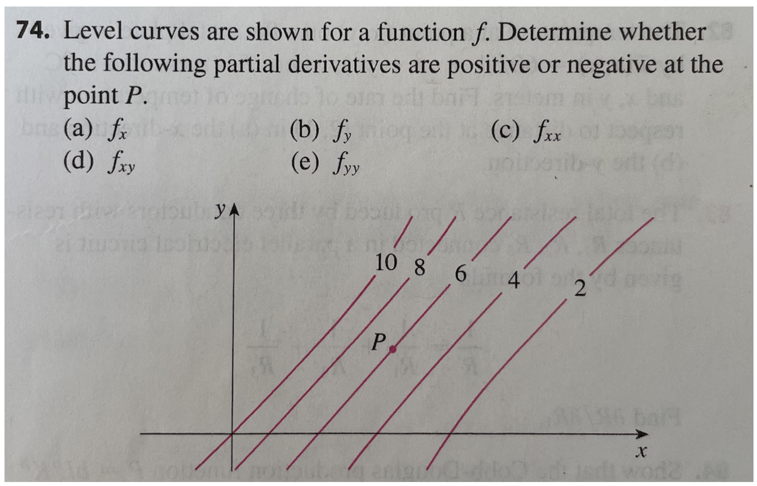
Problem 1. Compute all second mixed partial derivatives.

(a) $f(x, y, z) = x^a e^y \sin z + \frac{xy}{z^2}$.

(b) $f(x, y) = \int_0^{xy} g(t)\,\mathrm{d}t$

Problem 2. Let $f(x, y, z) = e^{xy} + \sqrt{z^3 + x^2} + z\sin(y)$. Compute $\frac{\partial^3 f}{\partial x \partial y \partial z}$.

Problem 3. (Textbook 14.3.74.)



74. Level curves are shown for a function f. Determine whether the following partial derivatives are positive or negative at the point P.

(a) f_x (b) f_y (c) f_{xx}

(d) f_{xy} (e) f_{yy}

Problem 4. (Textbook 14.3.88) The ideal gas equation is

$$PV = nRT.$$

Show that

$$\frac{\partial P}{\partial V}\frac{\partial V}{\partial T}\frac{\partial T}{\partial P} = -1.$$

2 Tangent Planes

Problem 5. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a nice function, and consider the graph of f, i.e. $z = f(x, y)$. What is the equation of the tangent plane to f at the point (x^*, y^*)?

Problem 6. Consider the unit sphere in 3 dimensions. What is the equation of the tangent plane at $(0, 0, 1)$? Figure this out two ways: (1) draw a picture and make a good guess, and (2) using the previous problem.

Problem 7. (Textbook 14.4.36) The wind-chill index is modeled by the function

$$W(v, T) = 13.12 + 0.6125T - 11.37v^{0.16} + 0.3965Tv^{0.16}.$$

The wind speed is measured at 26 km/h with possible error ± 2 km/h. The temperature is measured at -11°C with possible error $\pm 1^{\circ}$C.

(a) Find the linear approximation at $v = 26$ and $T = -11$.

(b) Use the linear approximation from (a) to estimate the maximum error in windchill index.

3 Chain Rule.

Problem 8. (Textbook 14.5.21) Let $T = \frac{v}{2u+v}$, $u = pq\sqrt{r}$, and $v = pr\sqrt{q}$. Compute

$$\frac{\partial T}{\partial p}, \frac{\partial T}{\partial q}, \frac{\partial T}{\partial r}.$$

Problem 9. Suppose $f(x, y) = y^2 \sin(x)$, $x = g(u, t, s)$, and $y = h(u, t, s)$. Let **p**=(1,2,3). Given the following values:

$$g(p) = \pi/3 \qquad h(p) = 4/3$$

$$\frac{\partial g}{\partial u}(p) = 1 \qquad \frac{\partial g}{\partial s}(p) = -1$$

$$\frac{\partial h}{\partial u} = u^2 + st \qquad \frac{\partial h}{\partial s} = ut$$

Compute $\frac{\partial z}{\partial u}(p)$ and $\frac{\partial z}{\partial s}(p)$.