

1 Level Sets

Problem 1. Let $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a function. Define the level set of f at c .

Problem 2. Draw level sets for the following functions. Also what are the equations of the level sets?

(a) $f(x, y) = x + y^2$ at $-1, 0, 1$.

(b) $f(x, y) = y - \cos(x)$ at $-7, 0, \pi$.

(c) $f(x, y, z) = x^2 - y^2 - z^2$ Show that the 0 level set is a cone. What are the surfaces corresponding to level sets at ± 1 ?

(d) $f(x, y, z) = \langle \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}, y \rangle$ What do the level sets of this curve look like for each level set (a, b) ?

2 Partial Derivatives

Problem 3. Textbook 14.3.53,56.

Problem 4. Prove that there is no nice function $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ is such that $f_x = 3y$, $f_y = 3x + z$ and $f_z = x$. (nice = twice continuously differentiable).

Problem 5. Textbook 14.3.74.

Problem 7. Textbook 14.3.88.

3 Tangent Planes

Problem 6. Textbook 14.4.36.