Math 53 Dis 108/111 Worksheet 4 (09/06)

Tom Schang

Problem 1. True or false? Let $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$, and $\vec{\mathbf{c}}$ all be vectors in $\mathbb{R}^3$.

(a) If $\vec{\mathbf{a}}$ is orthogonal to $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ then $\vec{\mathbf{a}}$ is orthogonal to $\vec{\mathbf{b}} + \vec{\mathbf{c}}$.

(b) If $\vec{\mathbf{a}}$ is orthogonal to $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ then $\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) \neq 0$

(c) If P_1 and P_2 are points in the plane $x + y + z = 1$, then $P_1 + P_2$ is also in the plane.

Problem 2. Lines, planes, normals.

(a) State what it means to be orthogonal in at least 2 ways.

(b) State what it means to be parallel in at least 2 ways.

(c) Let $ax + by + c = 0$ be a line. What is the parametric equation of the line? What is a normal vector for the line? Show that the difference between any two points on the line is orthogonal to the normal vector.

(d) Let L be the line through (x_1, y_1, z_1) and (x_2, y_2, z_2). What is the parametric equation of the line? What are the symmetric equations of the line? What is the vector equations of the line? What are two distinct normal vectors to the line?

(e) Let $ax + by + cz + d = 0$ be a plane. What is a normal vector for the plane? What is the vector equation for the plane? Show that the difference between any two points in the plane is orthogonal to the normal vector.

(f) Find the equation of a plane orthogonal to the line from (d).

Problem 3. Suppose you are travelling up a mountain along the curve

$$\langle (1 + t^2) \cos t, (1 + t^2) \sin t, -t^2 \rangle$$

for $-2\pi \leqslant t \leqslant 2\pi$.

(a) At what time are you travelling the fastest?

(b) At what times are you travelling directly northwest?

(c) Challenge: show that you are travelling on the surface of a cone.

Problem 4. Textbook problem 12.5.81.

Problem 5.

(a) Let g be an arbitrary function $g : \mathbb{R}^2 \to \mathbb{R}$. Define the level set at c of g.

(b) Let

$$f(x, y) = x + y^2$$

Draw the level sets for $f(x, y) = -1$, $f(x, y) = 0$, and $f(x, y) = 1$. Include the equation of at least one level set.