Math 53 Dis 108/111 Worksheet 15 (11/20)

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1 Basic Skills

I. State Stokes' theorem (and know how to compute all the quantities).

II. State Gauss' divergence theorem (and know how to compute all the quantities involved).

III. Remember/State the 2-d equivalents (Green's theorem versions 1 and 2).

2 Problems Types

I. Use Gauss' divergence theorem to switch between an integral over a boundary surface to an integral over a volume (and vice versa).

II. Use Stokes' theorem to switch between an integral over a boundary curve to an integral over a surface (and vice versa).

3 Practice

Problem 1. (Textbok 16.Review.31) Verify that Stokes' theorem is true for the vector field $\mathbf{F} = \langle x^2, y^2, z^2 \rangle$ where S is the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the xy-plane (with the upward orientation).

Problem 2. (Textbook 16.Review.35) Verify that the divergence theorem is true for the vector field $\mathbf{F} = \langle x, y, z \rangle$ where E is the unit ball $|\mathbf{x}| \leqslant 1$.

Problem 3. Suppose u, v are two functions taking $\mathbb{R}^3 \to \mathbb{R}$. Suppose that $v = 0$ on the set $|\mathbf{x}| = 1$ (note that $\mathbf{x}$ represents a *point* in $\mathbb{R}^3$, not just the x-coordinate). Show that

$$\iiint_{|\mathbf{x}| \leqslant 1} v \nabla^2 u \, \mathrm{d}V = -\iiint_{|\mathbf{x}| \leqslant 1} \nabla v \cdot \nabla u \, \mathrm{d}V.$$

[Hint: what is $\nabla \cdot (v \nabla u)$?]