MATH 53 Quiz 8 (10/18) **Name:**

Please write legibly and explain your work clearly. Answers without explanations may receive less (or no) credit.

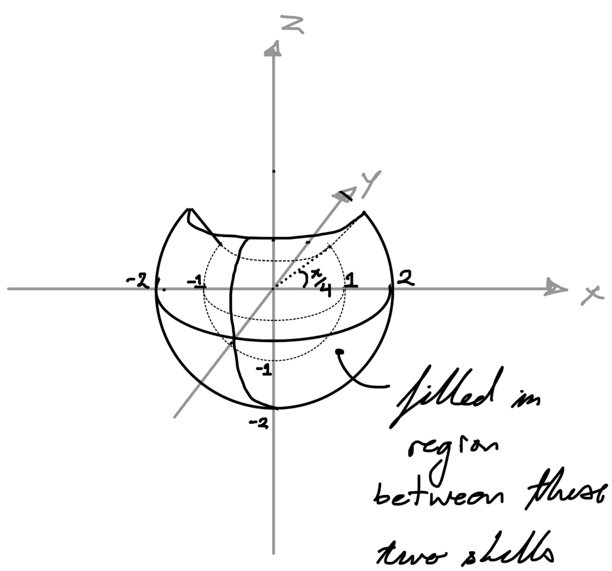
Problem 1. (4 points) What is the volume of the region R that is bounded by the surfaces $z = 1 - (y-1)^2$, $x = y$, $x - 3 = y$, and $z = 0$? Evaluate using a triple integral.

Solution: Because we want the volume between $z = 1-(y-1)^2$ and $z = 0$, we want to know the region where $1-(y-1)^2 \geqslant 0$, which is $0 \leqslant y \leqslant 2$. We are then given x bounds in terms of y and z bounds in terms of y, so we can write

$$\begin{aligned}\int_0^2 \int_y^{y+3} \int_0^{1-(y-1)^2} 1 \,\mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y &= \int_0^2 \int_y^{y+3} 1-(y-1)^2 \,\mathrm{d}x\,\mathrm{d}y \\ &= \int_0^2 \int_y^{y+3} 2y - y^2 \,\mathrm{d}x\,\mathrm{d}y \\ &= \int_0^2 (2y-y^2)(y+3-y)\,\mathrm{d}y \\ &=3\int_0^2 2y - y^2\,\mathrm{d}y = 3\left(y^2 - \frac{y^3}{3}\right)\Big|_0^2 = 3(4-\frac{8}{3}) = 4\end{aligned}$$

Problem 2. (4 points) Sketch the region given by the spherical coordinates $\cos\phi \leqslant \sqrt{2}/2$, $\pi \leqslant \theta \leqslant 2\pi$ and $1 \leqslant r \leqslant 2$. Label and explain your drawing.

Solution: This region has $\phi \geqslant \pi/4$, it has $y < 0$ (due to the theta), and it is contained between the shell $r = 1$ and the shell $r = 2$.



Problem 3. (4 points) Compute the integral.

$$\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{0}^{\sqrt{x^2+y^2}} (x^2 + y^2)\,\mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y$$

Solution: We will use cylindrical coordinate $x = r\cos\theta, y = r\sin\theta, z = z$ to turn this into

$$\int_{0}^{2\pi} \int_{0}^{2} \int_{0}^{r} r^2\ r\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta = 2\pi \int_{0}^{2} r^3 z\Big|_{z=0}^{z=r} \mathrm{d}r = 2\pi \int_{0}^{2} r^4\,\mathrm{d}r = \frac{2\pi}{5} r^5\Big|_{0}^{2} = \frac{64\pi}{5}.$$