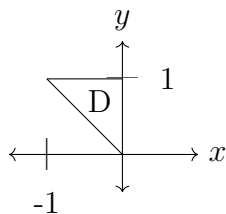**MATH 53 Quiz 6 (10/13)**   **Name:**

Please write legibly and explain your work clearly. Answers without explanations may receive less (or no) credit.

Problem 1. (4 points)

Write the correct bounds of integration for the region D for both integrals (note the order of integration).

$$\int \int f(x,y)\,\mathrm{d}y\,\mathrm{d}x \qquad\qquad \int \int f(x,y)\,\mathrm{d}x\,\mathrm{d}y$$

Solution:

$$\int_{-1}^{0} \int_{-x}^{1} f(x,y)\,\mathrm{d}y\,\mathrm{d}x \qquad\qquad \int_{0}^{1} \int_{-y}^{0} f(x,y)\,\mathrm{d}x\,\mathrm{d}y$$

Problem 2. (4 points) What is the area of the region enclosed by $y = x^3$ and $y^2 = \sqrt{32x}$?

Solution: The points of intersection are when $x^6 = 32x$, which occurs when $x = 0$ and when $x^5 = 32 \implies x = 2$. Thus we compute

$$\int_{0}^{2} \int_{x^3}^{\sqrt{32x}} \mathrm{d}y\,\mathrm{d}x = \int_{0}^{2} y\big|_{x^3}^{\sqrt{32x}}\,\mathrm{d}x = \int_{0}^{2} \sqrt{32x} - x^3\,\mathrm{d}x = \left[\frac{2}{3}\sqrt{32x^3} - \frac{x^4}{4}\right]_{0}^{2} = \frac{32}{3} - \frac{16}{4} = \frac{32-12}{3} = \frac{20}{3}$$

Problem 3. (4 points) Evaluate the integral

$$\int_0^2 \int_{x/2}^1 \frac{e^y}{y} \, dy \, dx.$$

Solution: The first step is to change the order of integration.

$$\int_0^2 \int_{x/2}^1 \frac{e^y}{y} \, dy \, dx = \int_0^1 \int_0^{2y} \frac{e^y}{y} \, dy \, dx$$

The next step is to integrate

$$\int_0^1 \int_0^{2y} \frac{e^y}{y} \, dy \, dx = \int_0^1 \frac{e^y}{y} [x]_0^{2y} \, dy = \int_0^1 2e^y \, dy = 2(e - 1)$$