MATH 53 Quiz 3 (09/15) **Name:** ____________

Please write legibly and explain your work clearly. Answers without explanations may receive less (or no) credit.

Problem 1. (4 points, no partial credit) Let $f(x, y) = \sin(x^2 - y^2)$. Compute f_x and f_{xy}.

$f_x =$ ____________ $f_{xy} =$ ____________

Solution:

$$f_x = 2x\cos(x^2 - y^2) \qquad f_{xy} = 4xy\sin(x^2 - y^2)$$

Problem 2. (4 points) Find an equation of the tangent plane to the surface $z = x/y^2$ at the point (-4,2,-1).

Solution: We know how to find tangent planes to graphs of functions. In this case, we have the function $f(x, y) = x/y^2$. The way to find the tangent plan is to compute

$$\frac{\partial f}{\partial x} = \frac{1}{y^2} \qquad \text{and} \qquad \frac{\partial f}{\partial y} = \frac{-2x}{y^3}.$$

This then tells us two tangent vectors on our surface:

$$\langle 1, 0, 1/(2^2)\rangle = \langle 1, 0, 1/4\rangle \qquad \text{and} \qquad \langle 0, 1, -2(-4)/(2^3)\rangle = \langle 0, 1, 1\rangle$$

Now we can find the normal vector to our plane

$$\langle 1, 0, 1/4\rangle \times \langle 0, 1, 1\rangle = \langle -1/4, -1, 1\rangle$$

and thus the equation of our plane is

$$\frac{-1}{4}(x - (-4)) - 1(y - 2) + 1(z - (-1)) = \frac{-1}{4}(x + 4) - (y - 2) + (z + 1) = 0.$$

Problem 3. Level sets.

(a) (1 point) Let g be an arbitrary function $g : \mathbb{R}^2 \to \mathbb{R}$. Define the level set at c of g.

Solution: the level set at c of g is the set of input values (x, y) such that $g(x, y) = c$. This can be written

$$\{(x, y) \in \mathbb{R}^2 : g(x, y) = c\}.$$

(b) (3 points) Let

$$f(x, y) = (1 + x^2)y$$

Draw the level sets for $f(x, y) = -1$ and $f(x, y) = 2$. Explain and label your drawing.

Solution: The level set at 2 corresponds to the equation $\frac{2}{1+x^2} = y$, which looks like a bell curve with value 2 at $x = 0$. For the level set at -1, the equation is $y = \frac{-1}{1+x^2}$.