MATH 54 Quiz 2 (09/08) **Name:**

Please write legibly and explain your work clearly. Answers without explanations may receive less (or no) credit.

Problem 1. (4 points) True or False?

T—F. There exist two vectors $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ in $\mathbb{R}^3$ so that $|\vec{\mathbf{u}}+\vec{\mathbf{v}}| = |\vec{\mathbf{u}}| + |\vec{\mathbf{v}}|$.

Solution: True. For example, take the vectors $\vec{\mathbf{u}} = \vec{\mathbf{v}} = \langle 1,0,0\rangle$. Then

$$|\vec{\mathbf{u}}+\vec{\mathbf{v}}| = |\langle 2,0,0\rangle| = 2 = |\langle 1,0,0\rangle| + |\langle 1,0,0\rangle| = |\vec{\mathbf{u}}| + |\vec{\mathbf{v}}|.$$

T—F. For any two vectors $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ in $\mathbb{R}^3$, $|\vec{\mathbf{u}}\times\vec{\mathbf{v}}| = |\vec{\mathbf{v}}\times\vec{\mathbf{u}}|$.

Solution: True. Using the fact that $\vec{\mathbf{u}}\times\vec{\mathbf{v}} = -\vec{\mathbf{v}}\times\vec{\mathbf{u}}$, we deduce that

$$|\vec{\mathbf{u}}\times\vec{\mathbf{v}}| = |-\vec{\mathbf{v}}\times\vec{\mathbf{u}}| = |\vec{\mathbf{v}}\times\vec{\mathbf{u}}|.$$

T—F. Two lines in $\mathbb{R}^3$ either intersect or are parallel.

Solution: False. For example, consider the x-axis and the line $x = 0, z = 1$ (i.e. the y-axis, but shifted up by 1 from the origin).

T—F. In $\mathbb{R}^3$, if two lines are parallel to a plane then the lines are parallel.

Solution: False. For example, consider the xy-plane, the x-axis as one line, and the y-axis as the other line.

Problem 2. (4 points) Suppose you throw a frisbee and that it follows the trajectory

$$\mathbf{r}(t) = \left\langle 4\ln(t+1),\ \frac{t^2}{12},\ \frac{-1}{2}(t-1)^2+2 \right\rangle, \quad 0 \leqslant t \leqslant 3.$$

What is the equation of the tangent line at the point where the frisbee is flying horizontally?

Solution: start by computing

$$\mathbf{r}'(t) = \left\langle \frac{4}{t+1},\ \frac{t}{6},\ -(t-1) \right\rangle.$$

This is travelling horizontally when $z'(t) = 0$, which is when $-(t-1) = 0 \implies t = 1$. Thus the parametric equation of the line is

$$\mathbf{r}(1) + t\mathbf{r}'(1) = \left\langle 4\ln 2, \frac{1}{12}, 2 \right\rangle + t\left\langle 2, \frac{1}{6}, 0 \right\rangle$$

Problem 3. Let $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$, and $\vec{\mathbf{c}}$ all be vectors in $\mathbb{R}^3$ such that

i. $\vec{\mathbf{a}}$ is orthogonal to $\vec{\mathbf{c}}$
ii. $\vec{\mathbf{b}}$ is orthogonal to $\vec{\mathbf{c}}$
iii. $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are not parallel.
iv. $\vec{\mathbf{a}} \bullet \vec{\mathbf{b}} \neq 0$.

(a) (1 point) What is the angle between $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$?

Solution: $\pi/2$, because they are orthogonal.

(b) (1 point) What are possible angles between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$?

Solution: $(0, \pi/2) \cup (\pi/2, \pi)$, because they are not orthogonal (hence the angle is not $\pi/2$), and because they are not parallel (hence the angle is not 0 or π).

(c) (2 points) Create example vectors $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$, and $\vec{\mathbf{c}}$ that satisfy properties (i) through (iv). Justify that your vectors satisfy each requirement.

Solution: three vectors that work are $\vec{\mathbf{a}} = \langle 1, 0, 0 \rangle$, $\vec{\mathbf{b}} = \langle 1, 1, 0 \rangle$, and $\vec{\mathbf{c}} = \langle 0, 0, 1 \rangle$. We verify the 4 conditions:

i. $\vec{\mathbf{a}} \cdot \vec{\mathbf{c}} = 1(0) + 0(0) + 0(1) = 0$ so they are orthogonal.
ii. $\vec{\mathbf{b}} \cdot \vec{\mathbf{c}} = 1(0) + 1(0) + 0(1) = 0$ so they are orthogonal.
iii. $\vec{\mathbf{a}} = k\vec{\mathbf{b}} \implies 1 = k$ and $0 = k$, which doesn't have a solution. Thus $\vec{\mathbf{a}} \neq k\vec{\mathbf{b}}$ for any k, so they are not parallel.
iv. $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 1(1) + 0(1) + 0(0) = 1 \neq 0$.