HW1 Rubric

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10.1

Key Concepts

1. Drawing graphs defined parametrically.
2. Recognizing important identities.
3. Substitutions and algebra.

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10.1.1-3 The expected strategy is to plug in some t-values and plot those points, together with lines interpolating between points.

10.1.5, 10.1.7 The expected strategy is to create an expression for t in terms of x or y and substitute in to the other equation. Sketching as in 10.1.1-3.

10.1.11-12 The expected strategy is to use a trig identity to directly give an expression for x and y. In particular, for 10.1.11,

$$x^2 + y^2 = \sin^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2} = 1$$

directly gives a cartesian expression for x and y.

An attempt to create an expression for t in terms of x or y will be accepted, but you should re-try these problems since such a solution may not receive full credit on an exam or quiz.

10.1.24 The expected strategy is to show me or describe key points from each graph that correspond to the parametric coordinates.

10.1.37-38 The expected strategy is to plot each of these curves and indicate different branches or key points, such as when $t \to \infty$. If there are no pictures and the results do not make sense, I cannot evaluate the attempt.

10.2

Key Concepts

1. Finding the tangents to a curve defined parametrically. [This may involve limits!]
2. Finding the concavity of a curve defined parametrically.

3. Computing the area under a curve defined parametrically.
4. Computing the arc length of a curve defined parametrically.

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10.2.1-5 The expected strategy is to use the identity

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

which requires computing dx/dt and dy/dt. For problems 3-5, one correct strategy is to use the point-slope formula.

10.2.11, 10.2.13 The expected strategy is to use the formula

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt}.$$

Then one may find zeroes of the resulting equation, and determine the sign in between zeroes to determine concavity. [If you forget concavity, please review 1-variable calculus!]

10.2.17-19 The expected strategy is to compute dy/dt and dx/dt. Zeroes of dy/dt **may** correspond to horizontal tangents. Zeroes of dx/dt **may** correspond to vertical tangents.

For 10.2.19: However, when both are zero for the same t^*, one has to take a limit as $t \to t^*$ to determine what the derivative at that value of t^* is. If I do not see a limit set up, then I will not give credit. [See example 2 in section 10.2 on page 651]

10.2.29 The expected strategy is to compute $\frac{dy}{dx}$ and to set the expression equal to 1/2 and solve for t.

10.2.30 The expected strategy is to compute $\frac{dy}{dx}$. Then, due to ambiguity of wording, I will accept either

1. Finding the slope at the point $(4, 3)$ and creating an equation for that tangent line.
2. Creating the point slope formula with symbolic derivative and determining whether the system of equations has solutions.

10.2.32-33 The expected strategy is to set up the integral

$$\int f(t)g'(t)dt$$

for appropriate f and g'.

10.2.41-44 The expected strategy is to set up the integral

$$\int \sqrt{(dy/dt)^2 + (dx/dt)^2}.$$

10.2.48 The expected strategy is to find two distinct t-values such that $x(t_1) = x(t_2)$ and $y(t_1) = y(t_2)$ (system of equations) to determine the bounds of integration. Then one is expected to use the integral formula for parametric arc length.

10.2.51-52 The expected strategy is to use the arc-length formula to find the distance travelled. To find the length of the curve, either algebraic manipulations or analysis of the periodicity of the parametric equations should be used to find new bounds of integration, at which point the same arc-length formula can be used.

10.2.53 The expected strategy is to use the arc-length formula and then do some algebra (the key step being a trig substitution) to get the desired formula.

10.2.73 The expected strategy is to break up the x and y coordinates into components, and then use trigonometric identities to create closed-form expressions. Pictures are instrumental in demonstrating understanding.

10.2.74 The expected strategy is to break up the region inside a semi-circle, the two regions under the involute, and the area in the silo (a circle). The area of the region under the involute can be computed using the parametric area-under-a-curve formula.