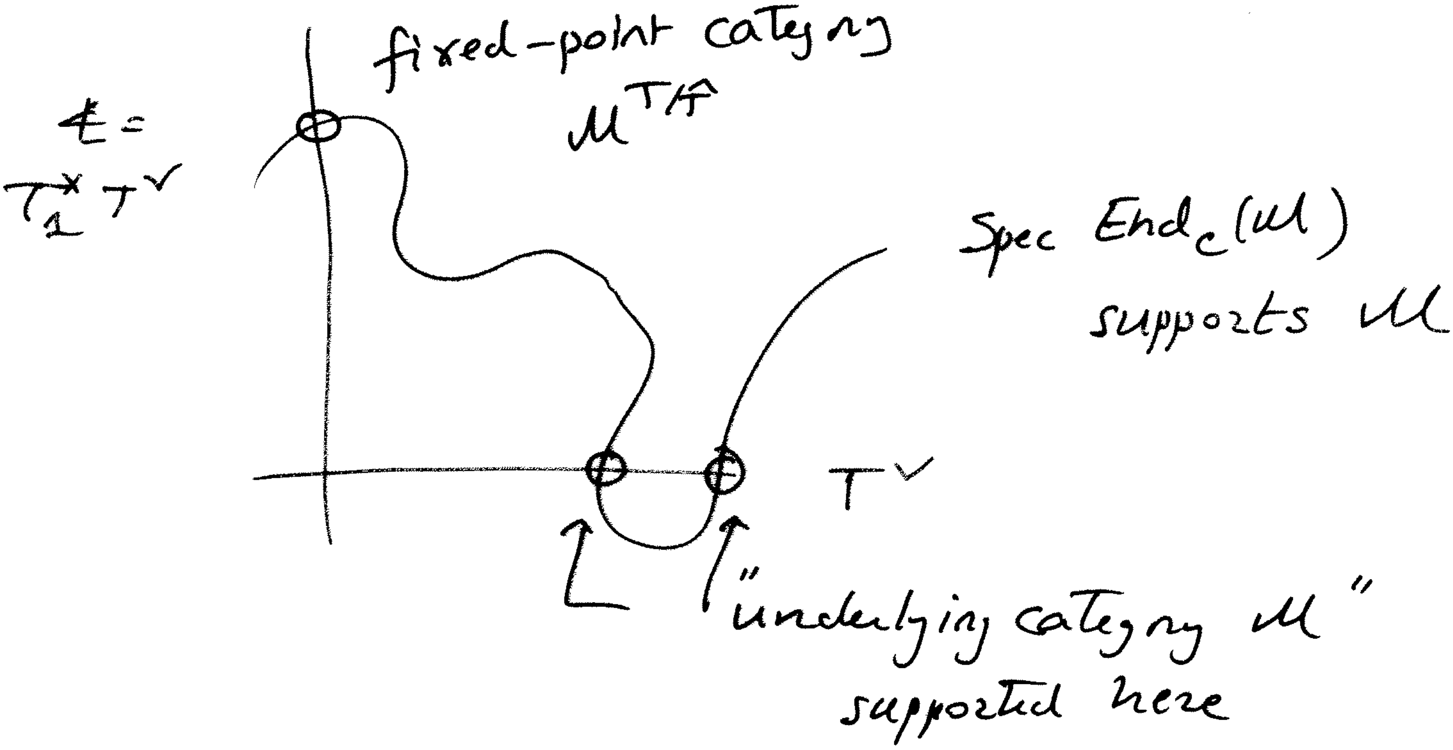
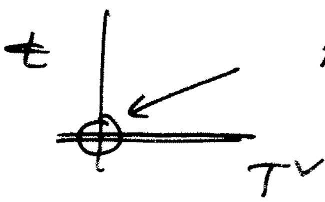
Semiclassical picture of a module category $\mathcal{M}$ over $(\mathrm{Coh}(T^\vee), \otimes) = \mathcal{C}$



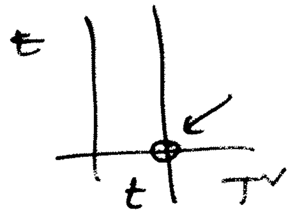
This is like the "singular support" or "characteristic cycle" of a $\mathcal{D}$-module

Examples:

$\mathcal{M} = \mathrm{Coh}(T^\vee)$:

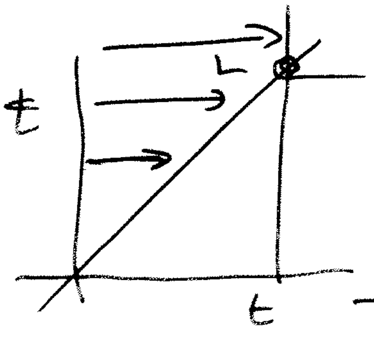


$\mathcal{M} = \mathrm{Vect}_t \quad t \in T^\vee$:



$\mathcal{M} = \mathrm{Vect}_\Delta$,

Casimir representation



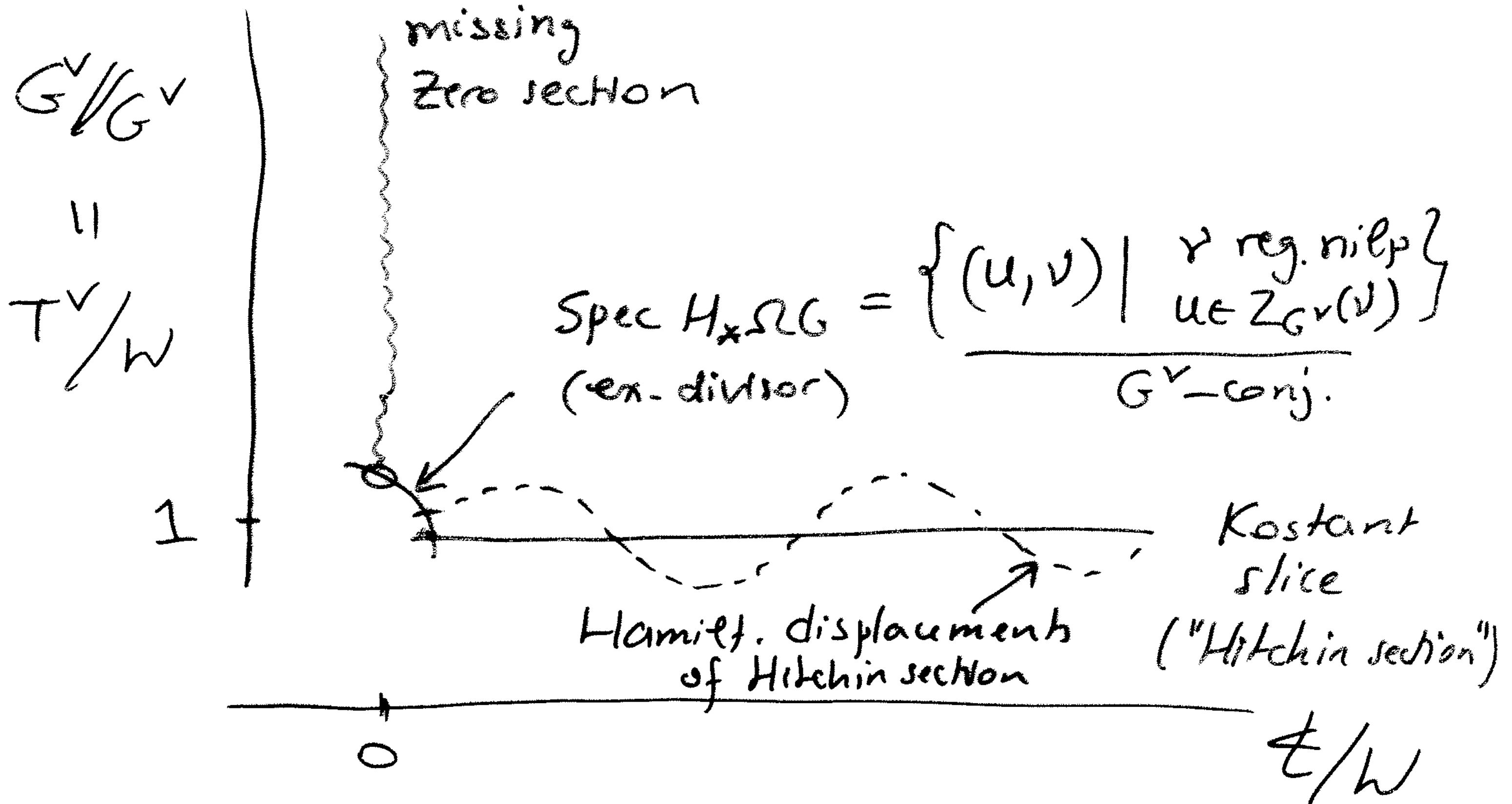
Note that it meets every Vect_t !

Bezrukavnikov-Finkelberg-Mirkovic Theorem I

$$T^*_{reg} G^\vee /\!/ G^\vee = \frac{\{ (g, \xi) \mid \xi \in (\mathfrak{g}^\vee)^*_{reg},\ g \in Z_{G^\vee}(\xi) \}}{G^\vee\text{-conjugation}}$$

$$\left(= \text{affine blow-up of } (T^\vee \times \mathfrak{t})/W \right)$$

$$= \operatorname{Spec} H^G_*(\Omega G).$$



$$\shortparallel$$

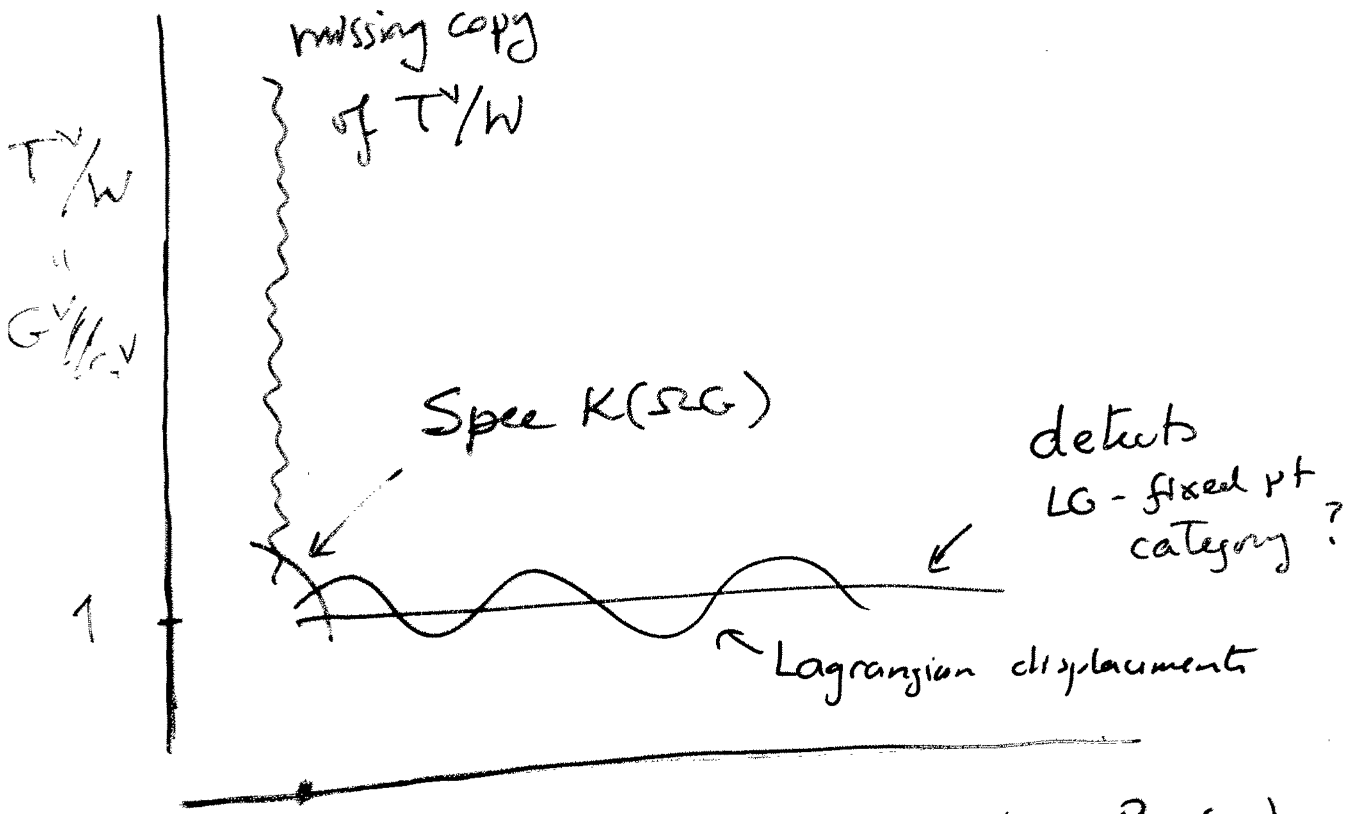
$$\operatorname{Spec} H^*_G(pt) = (\mathfrak{g}^\vee)^*_{reg} / G^\vee$$

Note: $\operatorname{Spec} H_*\Omega G \cong$ centralizer of any fixed principal nilpotent elt.

Bezrukavnikov – Mirković – Finkelberg Theorem II

Spec $K^G(\Omega G)$ = affine blow-up of $(T^\vee \times T)/W$

always Poisson, sometimes symplectic



T/W = Spec $K_G(pt)$ = Rep(G)

Recall Verlinde picture:

* Level $\Rightarrow$ isogeny $T \to T^\vee$
  Its graph is a Lagrangian in $(T \times T^\vee)/W$.
* Can be deformed by adding dF for F a function on T/W.