Math185 – Midterm 2

25 points, 75 minutes. Closed book, no notes.

Question 1, 4 pts

For the following functions, locate the singularities, determine their type and find the residue if appropriate:

$$(a)\frac{e^z+1}{e^z-1} \qquad (b)\sin^2(1/z)$$

Question 2, 5 pts

For the function $f(z) = \dfrac{1}{z(z^2-1)}$, find two Laurent expansions centered at $z = 0$ and converging respectively in the annuli $0 < |z| < 1$ and $|z| > 1$.

Determine the integral over the circle C of radius 17 centered at $z = 0$,

$$\oint_C \frac{dz}{z^4(z^2-1)}.$$

Question 3, 5 pts

Show that

$$\int_0^{2\pi} \frac{d\theta}{5 - 3\cos\theta} = \frac{\pi}{2}.$$

Question 4, 6 pts

By using a keyhole contour and the logarithm as an auxiliary function, or otherwise, show that

$$\int_0^{\infty} \frac{x dx}{(x+1)(x^2+x+1)} = \frac{\pi}{3\sqrt{3}}$$

Note: $\exp(\pm 2\pi i/3) = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$

Question 5, 5pts

Let $f(z)$ be a holomorphic function in a disk of radius $R + \varepsilon$ centered at $z = 0$ $(R, \varepsilon > 0)$. Using Cauchy's formula, prove that f has a Taylor series expansion centered at $z = 0$ which converges absolutely in the disk of radius R.