Math 185, Fall 2014: Third exam, 11/6

25 points, 75 minutes

You must justify your answers.

Question 1 (3+3)

Locate and classify all singularities for the following functions, and find the residues at the isolated singularities:

$$\text{(a) } \frac{z}{\sin z} \qquad \text{(b) } \frac{1}{e^{1/z}-1}$$

Caution. One of these functions also has a non-isolated singularity; do not attempt to compute a residue *there*.

It may help your residue calculation at a singular point p to write $z = p + w$, and work with w.

Question 2 (5)
Write down a proof of the Fundamental Theorem of Algebra that uses complex methods.
If you use a theorem from complex analysis, such as the Maximum Modulus Theorem or Liouville's theorem, include in outline an argument for it, starting from Cauuchy's Integral Formulas.

Question 3 (7)
Evaluate

$$\int_0^{2\pi} \frac{1+\cos\theta}{13-12\sin\theta} d\theta$$

Question 4 (7)
By a residue calculation of a contour integral along a carefully chosen sector, determine that

$$\int_0^\infty \frac{x^2 dx}{(x^4+4)^2} = \frac{\pi}{64}$$

Remark: More than once choice of sector will work, but there is a best choice