Complex Analysis II

Outline

Meromorphic functions

Power series

- McLaurin series
- Laurent series

Branch cuts; the logarithm

Analytic continuation

- Schwarz reflection

Considering $\frac{f'(z)}{f(z)}$

- The argument principle
- Rouche's theorem

The residue theorem; definite integrals on the real line

Problems

5.7.9 – Spring 2001 18 Let f be an entire function such that

$$\int_0^{2\pi} |f(re^{i\theta})|^2 \, d\theta \leq Ar^{2k}, \qquad (0 < r < \infty),$$

where k is a positive integer and A is a positive constant. Prove that f is a constant multiple of the function z^k.

5.6.2 – Spring 1988 4 True or false: A function $f(z)$ analytic on $|z-a| < r$ and continuous on $|z-a| \leq r$ extends, for some $\delta > 0$, to a function analytic on $|z-a| < r+\delta$? Give a proof or a counterexample.

5.7.6 – Summer 1977 2 Let f be continuous on $\mathbb{C}$ and analytic on $\{z : \mathrm{Im}(z) \neq 0\}$. Prove that f must be analytic on $\mathbb{C}$.

5.6.20 – Fall 1992 9 Let the function f be analytic in the region $|z| > 1$ of the complex plane. Prove that if f is real valued on the interval $(1, \infty)$ of the real axis, then f is also real valued on the interval $(-\infty, -1)$.

5.8.25 – Summer 1981 19 Prove that the number of roots of the equation $z^{2n} + \alpha^2 z^{2n-1} + \beta^2 = 0$ (n a natural number, α and β real and nonzero) that have positive real part is n if n is even and $n-1$ if n is odd.

5.8.7 – Fall 1983 6 Consider the polynomial

$$p(z) = z^5 + z^3 + 5z^2 + 2.$$

How many zeros (counting multiplicities) does p have in the annular region $1 < |z| < 2$?

5.11.23 – Spring 2001 4 Evaluate

$$\int_0^\infty \frac{1}{1+x^5} dx.$$

5.7.4 – Fall 1999 16 For $0 < a < b$, evaluate the integral

$$I = \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{|ae^{i\theta} - b|^4} \, d\theta.$$