Outline

Explicit solutions
- Linear ODEs
  - Particular and homogeneous solutions, superposition, initial conditions
  - Second order constant coefficient linear ODEs
  - First order systems of linear ODEs
- Separation of variables
- Integrating factors

Local existence and uniqueness
- Picard-Lindelof
- Inverse and implicit function theorems

Global behavior
- Blowup
- Gronwall's inequality

Problems

Spring 2008 4A Find the solution of the differential equation

\[ y'' - 2y' - y = e^{-x} \]

satisfying \( y(0) = y'(0) = 0 \).

Fall 2021 6A Solve the initial value problem

\[ y' = Ay = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}, \quad y(0) = y_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \]

Fall 2021 1A
- a) Find a particular solution \( y_1(t) \) of

\[ y' = y^2 - ty + 1 \]

- b) Find the general solution. (The solution may be expressed using a definite integral)

3.1.3 - Fall 1993 14 Let \( n \) be an integer larger than 1. Is there a differentiable function on \([0, \infty)\) whose derivative equals its \( n \)th power and whose value at the origin is positive?
3.1.10 - Fall 1982 1 \ Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous nowhere vanishing function, and consider the differential equation
\[
\frac{dy}{dx} = f(y)
\]

1. For each real number $c$, show that this equation has a unique continuously differentiable solution $y = y(x)$ on a neighborhood of 0 which satisfies the initial condition $y(0) = c$.

2. Deduce the conditions on $f$ under which the solution $y$ exists for all $x \in \mathbb{R}$, for every initial value $c$.

Summer 1982 6 Suppose $f$ is a differentiable function from the reals into the reals. Suppose $f'(x) > f(x)$ for all $x \in \mathbb{R}$, and $f(x_0) = 0$. Prove that $f(x) > 0$ for all $x > x_0$. 