Differential Equations

Outline

Explicit solutions

- Linear ODEs
  - Particular and homogeneous solutions, superposition, initial conditions
  - Second order constant coefficient linear ODEs
  - First order systems of linear ODEs
- Separation of variables
- Integrating factors

Local existence and uniqueness

- Picard-Lindelof
- Inverse and implicit function theorems

Global behavior

- Blowup
- Gronwall's inequality

Problems

Spring 2008 4A Find the solution of the differential equation

$$y'' - 2y' - y = e^{-x}$$

satisfying $y(0) = y'(0) = 0$.

Fall 2021 6A Solve the initial value problem

$$y' = Ay = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}, \; y(0) = y_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Fall 2021 1A

- a) Find a particular solution $y_1(t)$ of

$$y' = y^2 - ty + 1$$

- b) Find the general solution. (The solution may be expressed using a definite integral)

3.1.3 - Fall 1993 14 Let n be an integer larger than 1. Is there a differentiable function on $[0, \infty)$ whose derivative equals its nth power and whose value at the origin is positive?

Differential Equations

3.1.10 - Fall 1982 1 Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous nowhere vanishing function, and consider the differential equation

$$\frac{dy}{dx} = f(y)$$

1. For each real number c, sow that this equation has a unique continuously differentiable solution $y = y(x)$ on a neighborhood of 0 which satisfies the initial condition $y(0) = c$.
2. Deduce the conditions on f under which the solution y exists for all $x \in \mathbb{R}$, for every initial value c.

Summer 1982 6 Suppose f is a differentiable function from the reals into the reals. Suppose $f'(x) > f(x)$ for all $x \in \mathbb{R}$, and $f(x_0) = 0$. Prove that $f(x) > 0$ for all $x > x_0$.