

- (20) 1. Put the following matrix in canonical form.

$$\begin{pmatrix} 3 & -2 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 1 \end{pmatrix}$$

(20) 2. Consider the system

$$X' = AX, \quad A = \begin{pmatrix} a & 0 & b \\ 0 & b & 0 \\ -b & 0 & a \end{pmatrix}$$

Sketch the regions in the  $ab$ -plane where this system has different types of phase portraits.

- (20) 3. a) Define topologically conjugate systems.  
b) Are the following two flows conjugate? If yes then find a conjugacy.

$$X' = \begin{pmatrix} 3 & 1 \\ 0 & -1 \end{pmatrix} X, \quad Y' = \begin{pmatrix} -3 & 2 \\ 0 & 1 \end{pmatrix} Y,$$

- (20) 4. a) Suppose  $X' = AX$  where  $A$  is a  $3 \times 3$  matrix whose eigenvalues are  $-2, \pm i$ . Describe the flow.
- b) What if the eigenvalues of  $A$  are  $\pm 2, 1$  ?

(20) 5. Consider the forced harmonic oscillator

$$x'' + 4x' + 4x = \sin 2t$$

Find its periodic solutions.