

Math 123

Final Exam

Fall 2007

No books, notes or calculators

Name:

Student ID:

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Problem	Grade
1	
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Total:	

- (25) 1. For the function $V(x, y) = 2x^2 - 4xy + y^4$ sketch the phase portrait of the gradient flow $X' = -\text{grad}V(X)$. Sketch the level surfaces of V on the same diagram. Find all the equilibrium points and determine their type.

- (25) 2. Let F be a smooth vector field in R^2 and $X \in R^2$ so that $X \in \omega(X)$. Show that X is either an equilibrium or it is on a closed orbit.

(25)

3. Let B be a ball in \mathbb{R}^2 . Let F be a smooth vector field in B that points inward along the boundary of the ball. Suppose that F has a single equilibria X_0 in B . Show that X_0 cannot be a saddle.

(25) 4. Sketch the phase portrait for the system of differential equations

$$\begin{cases} x' = \sin x \\ y' = \sin y \cos y \end{cases}$$

- (25) 5. Study the stability and the asymptotic stability of the zero equilibria for the system

$$\begin{cases} x' = y - xz^2 \\ y' = -x - yz^4 \\ z' = -z^3 \end{cases}$$

- (25) 6. State the local existence and uniqueness theorem for ordinary differential equations.

(25) 7. State and prove Gronwall's inequality.

(25) 8. Discuss the local and global behavior of solutions to

$$\begin{cases} r' = 2r - r^2 + a \\ \theta' = 1 \end{cases}$$

at all of the bifurcation values.