MATH 152 MIDTERM
Thursday, September 22, 2016

This exam has 4 problems on 10 pages, including this cover sheet and extra blank space at the end. The only thing you may have out during the exam is the exam itself and one or more writing utensils. You have 80 minutes to complete the exam.

DIRECTIONS

- Be sure to carefully read the directions for each problem.
- All work to be graded must be done on this exam. If you need extra space to finish a problem, make a note and finish it on the last page. This should really only happen if you mess up and want to start over without erasing.
- Grids are provided for graphing on some problems; you will need to determine where to place the axes. Make sure everything is clearly labeled.
- Good luck; do the best you can!

Problem	Max	Score
1	20	
2	30	
3	30	
4	20	
Total	100	

Problem 0: Take a deep breath – you've got this!

1. Prove the following special case of Lemma 1.3 from our textbook, which we have used many times.

   Suppose U and V are two different points in the plane, and Q is a point on the line segment $\overline{UV}$, with $Q \neq U$ and $Q \neq V$. Further assume that the line connecting U and V is not vertical and it is determined by the equation $y = mx + k$ for some constants m and k. Let $\ell : \mathbb{R}^2 \to \mathbb{R}$ be a linear function in two variables.

   Prove that if $\ell(U) \leq \ell(V)$, then $\ell(U) \leq \ell(Q) \leq \ell(V)$. (You may simply cite any needed theorems from 151 material, as well as theorems on basic arithmetic with inequalities, without proving these facts during the exam. You should cite the actual statements of the results, not theorem/lemma numbers.)

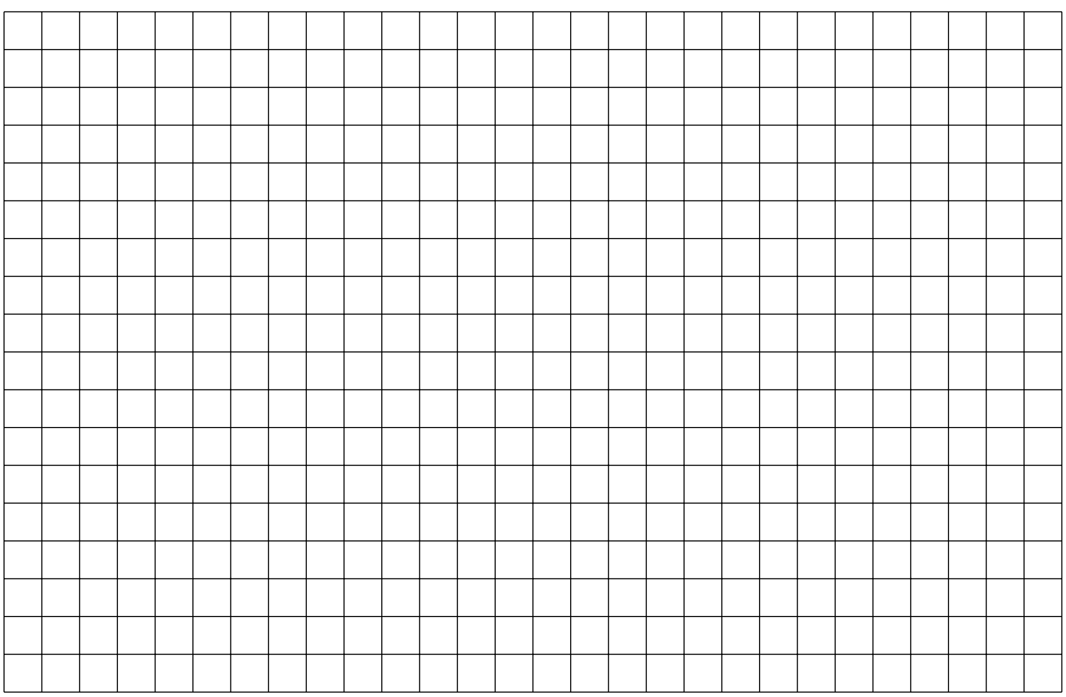
Extra space for problem 1 if needed.

2. Let $\ell(x, y) = 3x - y$. Determine, with justification, the **integer lattice points** at which ℓ achieves its maximum and minimum on the region determined by the intersection of the closed half-planes determined by the following inequalities. As part of your solution, provided a clearly labeled graph on the graph paper on the next page, with everything drawn to scale.

$$\begin{aligned} 4x + 2y &\geq 8 \\ y - 2 &\leq x \\ x &\leq 7 \end{aligned}$$

3. Of the two following quadratic equations, one is a "degenerate hyperbola" and one is actually a hyperbola. For the degenerate hyperbola, write its equation in what you might call normal form, and do your best to figure out what geometric object it is and graph it. For the real hyperbola, write its equation in normal form, and on your graph, clearly label any relevant features (center, foci, asymptotes, vertices, major and minor semi-axes, etc.) and give their coordinates and/or equations, as appropriate.

   (a) $25x^2 - 49y^2 + 50x + 196y + 1054 = 0$



Directions repeated: Of the two following quadratic equations, one is a “degenerate hyperbola” and one is actually a hyperbola. For the degenerate hyperbola, write its equation in what you might call normal form, and do your best to figure out what geometric object it is and graph it. For the real hyperbola, write its equation in normal form, and on your graph, clearly label any relevant features (center, foci, asymptotes, vertices, major and minor semi-axes, etc.) and give their coordinates and/or equations, as appropriate.

(b) $4x^2 - 9y^2 - 16x - 18y + 7 = 0$

4. Using the geometric definition*, find the equation of the parabola whose focus is $(14, 6)$ and whose vertex is $(2, 6)$. Your equation should be in $Ax^2 + Cy^2 + Dx + Ey + F = 0$ form. (Hint: first make a sketch and determine the directrix.)

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