# Examples to practice for the final

First, the basics. Give lots of examples of each of these; then keep them in mind later for the more challenging give an example problems. You don't need to invent anything exotic, just recall the examples we have discussed.

- Fields
- Integral domains which are not fields
- Commutative rings with unity which are not integral domains
- Noncommutative rings
- Rings without unity (both finite and infinite)
- Ideals of $\mathbb{Z}_n$ (prime, maximal, or neither)
- Ideals of $\mathbb{Z}_m \times \mathbb{Z}_n$ (prime, maximal, or neither)
- Ideals of $F[x]$ (prime, maximal, or neither)
- Ring homomorphisms
- Abelian groups and their normal subgroups
- Nonabelian groups and their normal subgroups
- Group homomorphisms

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Exam-style give an example problems. If you can give multiple answers, that will be good practice.

1. An extension field of $\mathbb{Q}$ which is not an algebraic extension.
2. A basis for $\mathbb{Q}(\sqrt[5]{17})$ as a vector space over $\mathbb{Q}$.
3. A basis for $\mathbb{Q}(\sqrt{2}, \sqrt{5})$ as a vector space over $\mathbb{Q}$.
4. An ideal of $\mathbb{Z}_3 \times \mathbb{Z}_4$ which is not a prime ideal.
5. A principal ideal of $\mathbb{Z}_3 \times \mathbb{Z}_4$ which is a prime ideal.
6. A maximal ideal of $\mathbb{R}[x]$.
7. A ring which has no proper nontrivial maximal ideals.
8. A ring $R$ which is an integral domain but not a field, and an ideal $I$ of $R$ such that $R/I$ is not a field.
9. A ring $R$ which is an integral domain but not a field, and an ideal $I$ of $R$ such that $R/I$ is a field.
10. A polynomial ring $R$ which is an integral domain, and an ideal $I$ of $R$ such that $R/I$ has zero divisors.
11. A ring $R$ which is not an integral domain, and an ideal $I$ of $R$ such that $R/I$ is an integral domain.
12. A subring of $\mathbb{Z}_2 \times \mathbb{Z}_4$ which is not an ideal.
13. Two nonisomorphic rings which each contain 3 elements.
14. A nontrivial ring homomorphism $\phi : \mathbb{Z}[x] \to \mathbb{Z}_3[x]$.
15. A nontrivial ring homomorphism $\phi : \mathbb{Z}[x] \to \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$.

16. A polynomial in $\mathbb{Z}[x]$ which has 4 terms and is irreducible by Eisenstein's Criterion.
17. A polynomial in $\mathbb{Z}[x]$ which has 4 terms and is irreducible, but no Eisenstein Criterion applies.
18. An irreducible quadratic polynomial in $\mathbb{Z}_5[x]$.
19. Positive integers $k$ and $n$ such that the congruence $12x \equiv k(mod n)$ has exactly 4 solutions in $\mathbb{Z}_n$.
20. A ring with characteristic 8 which has a proper subring isomorphic to $\mathbb{Z}_8$.
21. Two different proper subgroups $A$ and $B$ of $D_4$ such that $A \lhd B$ and $B \lhd D_4$, but $A \ntriangleleft D_4$.
22. Two different subgroups of $\mathbb{Z}_2 \times \mathbb{Z}_4$, each of which is isomorphic to $\mathbb{Z}_4$.
23. Two subgroups $A$ and $B$ of $G = \mathbb{Z}_2 \times \mathbb{Z}_4$ such that $G/A \cong \mathbb{Z}_4$ and $G/B \cong \mathbb{Z}_2 \times \mathbb{Z}_2$.

See old handouts, plus the midterm and practice midterm for more practice on earlier material.