Math 113 Name: Kelli

MATH 113, QUIZ #7
THURSDAY, OCTOBER 8

1. (3 points each) GIVE AN EXAMPLE – For each of the following problems, give a SPECIFIC example of the algebraic object. No justification is required, but be sure that your choices clearly meet the conditions required.

   (a) Two nonisomorphic index 2 subgroups of D_8.

   $\langle r \rangle$, $\langle r^2, s \rangle$

   (b) A group G and a subgroup $H \leq G$ so that $|H| = 6$ and there are 7 left cosets of H in G.

   $G = \mathbb{Z}_{42}$, $H = \langle 7 \rangle$

   (c) An infinite group G and an infinite subgroup $H \leq G$ such that there are infinitely many left cosets of H in G.

   $G = \mathbb{R}$ | $G = GL(3, \mathbb{R})$

   $H = \mathbb{Z}$ | $H =$ diagonals in G

   (d) A nonabelian group containing at least 50 elements, all of which have order 1, 2, 3, or 6.

   $S_3 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3$ or $S_3 \times S_3 \times S_3$

   (e) Two nonisomorphic order 12 subgroups of $\mathbb{Z}_{36} \times \mathbb{Z}_{24}$.

   $\langle (3,0) \rangle \cong \mathbb{Z}_{12} \cong \mathbb{Z}_3 \times \mathbb{Z}_4$

   $\langle 6 \rangle \times \langle 12 \rangle \cong \mathbb{Z}_6 \times \mathbb{Z}_2$.

(f) A subgroup H of $\mathbb{Z}_4 \times \mathbb{Z}_8$ which is NOT of the form $H = A \times B$, where $A \leq \mathbb{Z}_4$ and $B \leq \mathbb{Z}_8$.

$H = \langle (1,1) \rangle$

(g) Two different products of cyclic groups (use $\mathbb{Z}_n$'s, not other cyclic groups you happen to know) which are isomorphic to $\mathbb{Z}_{36} \times \mathbb{Z}_{50}$. (Not including the given product, and don't just reorder factors.) $2^2 \cdot 3^2 \cdot 2 \cdot 5^2$

$\mathbb{Z}_4 \times \mathbb{Z}_9 \times \mathbb{Z}_{50}$, $\mathbb{Z}_{36} \times \mathbb{Z}_2 \times \mathbb{Z}_{25}$

(h) A group with at least 10 different elements of order 5.

$\mathbb{Z}_5 \times \mathbb{Z}_5$

2. (6 points) The following table is a group table for a group G of order 8 of an isomorphism type we have not encountered yet. (It's called the quaternion group.) Show that G is not isomorphic to any abelian group of order 8 or to D_4 and justify your answer.

	1	-1	i	$-i$	j	$-j$	k	$-k$
1	1	-1	i	$-i$	j	$-j$	k	$-k$
-1	-1	1	$-i$	i	$-j$	j	$-k$	k
i	i	$-i$	-1	1	k	$-k$	$-j$	j
$-i$	$-i$	i	1	-1	$-k$	k	j	$-j$
j	j	$-j$	$-k$	k	-1	1	i	$-i$
$-j$	$-j$	j	k	$-k$	1	-1	$-i$	i
k	k	$-k$	j	$-j$	$-i$	i	-1	1
$-k$	$-k$	k	$-j$	j	i	$-i$	1	-1

Table not symmetric over diagonal
$\Rightarrow$ not abelian.

How many elements ~~are their own squ~~ have $x^2 = e$?

D_4 : 5 (4 reflections + 180 rot) This : only 2 (two 1's on diag)