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MATH 113, QUIZ #6
October 6

1. List all of the isomorphism types for abelian groups of order 48, using FTFGAG form (i.e. powers of primes form).

		largest order of elt	# self inverse
1	$\mathbb{Z}_3 \times \mathbb{Z}_{16}$	48	
2	$\mathbb{Z}_3 \times \mathbb{Z}_8 \times \mathbb{Z}_2$	24	
3	$\mathbb{Z}_3 \times \mathbb{Z}_4 \times \mathbb{Z}_4$	12	$e \times \{0,2\} \times \{0,2$
4	$\mathbb{Z}_3 \times \mathbb{Z}_4 \times \mathbb{Z}_2 \times \mathbb{Z}_2$	12	$e \times \{0,2\} \times \{0,1$ $\times \{0,1$
5	$\mathbb{Z}_3 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$.	6	

2. This is a continuation of the previous problem. If you had a mystery abelian group $= M$ with 48 elements, how would you determine which of the above groups it was isomorphic to? Be sure your answer is complete enough to distinguish between any two of the types. (You can have multiple steps if necessary.)

Really depends on what info you know. One reasonable approach → try to figure out the largest order of an element in M.

If it's 48, 24, or 6, we are done (type 1, 2, 5 respectively). If it's 12, try counting how many elements are their own inverses → Type 3 has 4 (including the identity) and type 4 has 8

3. Prove or disprove: Let G and H be groups. If $B \leq G$ and $C \leq H$, then $B \times C \leq G \times H$.

- Clearly $B \times C \subseteq G \times H$.
- $B \times C$ has identity (e_G, e_H).
- If $(b, c) \in B \times C$, then $(b,c)^{-1}$ exists in $G \times H$, and we have $(b,c)^{-1} = (b^{-1}, c^{-1})$, which is in $B \times C$, since B and C both contain inverses of their elements.
- Let $(b_1, c_1), (b_2, c_2) \in B \times C$. Then $(b_1, c_1) \cdot (b_2, c_2) = (b_1 b_2, c_1 c_2) \in B \times C$ by the closure of B & C,

so $B \times C \leq G \times H$

4. Prove or disprove: If A is a subgroup of $G \times H$ (where G and H are groups, of course), then A must be of the form $A = B \times C$, where $B \leq G$ and $C \leq H$.

Let $G = H = \mathbb{Z}$.

Then $\langle (1,1) \rangle$ cannot be written as a product $B \times C$.