

2. Show very brief justification for each of the following T/F questions.

(a) **TRUE** **FALSE** The element $\bar{2}$ has order 2 in \mathbb{Z}_n whenever n is even.

in \mathbb{Z}_6 $|\bar{2}| = 3$

(b) **TRUE** **FALSE** If $G = \{e, x, x^2, \dots, x^{23}\}$ is a cyclic group of order 24, then G has exactly one subgroup of order 4.

From thm: $4|24$ and G is cyclic, so there is exactly one order 4 subgroup.

(c) **TRUE** **FALSE** In S_8 , the element $(1, 2)(1, 3, 4, 6)(2, 8)$ has order 4. (This is in cycle notation.)

~~$= (1, 6, 2, 8)$~~
 $= (1, 3, 4, 6, 2, 8)$
 order 6

(d) **TRUE** **FALSE** Every cyclic group of order 48 has at least 5 different generators.

$\gcd(k, 48) = 1$

$k = 1, 5, 7, 11, 13$ etc.

(e) **TRUE** **FALSE** Every dihedral group D_n with $n \geq 3$ has a subgroup containing exactly half its elements.

rotations

3 each
 → (answer)
 → 2 work