Math 113 Name: Kelli

MATH 113, QUIZ #4

September 17

1. Suppose that H and K are two subgroups of a group G. Prove that the intersection $H \cap K$ is also a subgroup of G.

We'll use the subgroup criterion.

- Clearly, $H \cap K$ is a subset of G, since H and K are.
- $H \cap K \neq \emptyset$ since $e \in H$ and $e \in K$ (subgroups always contain the identity), so $e \in H \cap K$.
- Suppose $a, b \in H \cap K$, so $a, b \in H$ and $a, b \in K$. By the subgroup criterion, we have $ab^{-1} \in H$ and $ab^{-1} \in K$, so $ab^{-1} \in H \cap K$.

Thus $H \cap K \leq G$.

2. Find an example that shows the union of two subgroups of a group G might not be a subgroup of G.

Let $G = \mathbb{Z}_6$, $H = \langle \bar{2} \rangle = \{\bar{0}, \bar{2}, \bar{4}\}$, $K = \langle \bar{3} \rangle = \{\bar{0}, \bar{3}\}$

Then

$H \cup K = \{\bar{0}, \bar{2}, \bar{3}, \bar{4}\}$ which is not closed under addition: $\bar{2} + \bar{3} = \bar{5} \notin H \cup K$.

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3. Give an example of each of the following. No justification is required.

(a) An infinite cyclic subgroup of $GL(2,\mathbb{R})$.

$$\left\langle \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \right\rangle = \left\{ \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix} : n \in \mathbb{Z} \right\}.$$

(b) A finite nontrivial subgroup of $GL(2,\mathbb{R})$.

$$\left\langle \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \right\rangle$$ order 2.

(c) A subgroup of $GL(2,\mathbb{R})$ which is not cyclic.

proper

$$\{ M \in GL(2,\mathbb{R}) : \det M = \pm 1 \}.$$

(d) Two different subgroups of $\mathbb{C}^* \times \mathbb{C}^*$ which are both isomorphic to $\mathbb{Z}_8$ (and of course, to each other).

$U_8 \times \{1\}$, $\{1\} \times U_8$, $\langle (e^{\frac{2\pi i}{8}}, e^{\frac{2\pi i}{8}}) \rangle$,

$\langle (e^{2\pi i/8}, -1) \rangle$ etc.

(e) A subgroup of $\langle \mathbb{R}^3, + \rangle$ which is isomorphic to $\mathbb{Z}$. = infinite cyclic subgroup.

$\{ (x, x, x) : x \in \mathbb{Z} \}$. or any $\langle (a, b, c) \rangle$ with a, b, c not all 0.

or $\{ (x, 0, 0) : x \in \mathbb{Z} \}$.