Math 113 Name: Kelli

MATH 113, QUIZ #3
SEPTEMBER 10

1. Find three proper nontrivial subgroups of $GL(\mathbb{R}, 3)$ (the group of 3×3 invertible matrices), no two of which are isomorphic. You don't need to show that each one is a subgroup, but briefly explain why they are not isomorphic.

Tons of examples → easiest to show $\not\cong$ using cardinality.

all uncountable w/ $\mathbb{R}$ entries:

- {invertible lower Δ}
- {lower Δ w/ 1's on diag}
- {invertible diagonal}

- $\left\langle \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \right\rangle$ order 2
- $\left\langle \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \right\rangle$ order 3
- $\left\langle \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \right\rangle$ infinite order

etc

2. Prove or give a counterexample (with explanation): if G is an abelian group and $\varphi : G \to H$ is a group homomorphism, then H is an abelian group.

False, see HW 3, problem 3b.

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3. Give an example of each of the following. No justification necessary.

(a) A cyclic group with at least 12 elements.

$\langle \mathbb{Z}_{12}, + \rangle$, $\langle \mathbb{Z}_n, + \rangle$ where $n > 12$, $\langle U_{12}, \cdot \rangle$, etc.

(b) A noncyclic group with 8 or fewer elements.

$\langle$ Klein 4-group from reading, op:

*	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

$\rangle$

(c) A group which is infinite and cyclic.

$\langle \mathbb{Z}, + \rangle$, $\langle \{6^n : n \in \mathbb{Z}\}, \cdot \rangle$

(d) An infinite nonabelian group.

$\langle GL(\mathbb{R}, 3)$, matrix mult $\rangle$

(e) An infinite abelian group that is not cyclic.

$\langle \mathbb{R}, + \rangle$, $\langle \{$diag matrices in $GL(\mathbb{R}, 3)\}$, matrix mult $\rangle$

$\langle \mathbb{C}, + \rangle$

$\langle \mathbb{R}^*, \cdot \rangle$